Southern Connecticut State University
School of Arts and Sciences
Department of Mathematics

MAT 450 Real Analysis

1 Catalog Description

Theoretical analysis of functions of one real variable; limits, continuity, differentiability, Riemann integral.

2 Purpose

MAT 450 Real Analysis or MAT 446 Advanced Calculus with Applications are required courses for the applied, pure, and teaching concentrations in the BS in mathematics. MAT 446 covers a variety of applied topics at a higher level than MAT 450, whereas MAT 450 is a "traditional" proof-based course in real analysis. MAT 450 presents the theoretical analysis of functions of one variable.

3 Credit

MAT 450 carries 3 semester-hours of college credit.

4 Prerequisites

"C-" or better in both MAT 250 and MAT 252.

5 Format

A lecture-recitation format is followed.

6 Course Objectives

Students completing MAT 450 should be able to:

1. Know the important theorems that underlie the principles of calculus, and how they are proven rigorously.
2. Prove statements about sequences and functions using $\epsilon - \delta$ definitions and the consequences of important theorems.
3. Provide examples and counterexamples which illustrate the importance of theorem hypotheses and differences between similar definitions.

7 Outline

1. Real numbers (15%)
   (a) Algebraic properties (field axioms)
   (b) Order properties and inequality theorems
   (c) Completeness property
   (d) Sequences and limits
      - Rigorous definition
      - Monotone sequences and the Monotone Convergence Theorem
   (e) Series and sequences of partial sums
   (f) Cauchy criterion for sequences and series
2. The topology of $\mathbb{R}$ (10%)
   (a) Open sets, closed set
   (b) Compact sets
      i. Sequential criteria (Bolzano-Weierstrass Theorem)
      ii. Equivalence to closed and bounded/Heine-Borel Theorem
3. Functions of one variable (25%)
   (a) Limits and continuity
   (b) Limit theorems with proofs
   (c) Continuous functions on a closed and bounded interval
      i. Extreme Value Theorem
      ii. Intermediate Value Theorem
   (d) Sequences of continuous functions
      i. Point-wise versus uniform convergence.
      ii. (Optional) History of continuity and convergence.
4. Derivatives of functions of one variable (20%)
   (a) Review definition
   (b) Proof of properties (suggested: sum, product, and chain rules)
   (c) Differentiability implies continuity
   (d) Increasing/decreasing functions, concavity (proofs)
   (e) Inverse functions and the derivative of the inverse
   (f) (Optional) Mean Value Theorem
5. Riemann integral (20%)
   (a) Upper and lower sums

(b) Proof of existence of definite integral
(c) Proofs of fundamental theorems
(d) Verification of properties
(e) (Optional) Comparison to other definitions of integral (Lesbesgue, Stieltjes)
(f) (Optional) Improper integrals

6. Taylor's Theorem (10%)
   (a) Remainder estimation (derivative and/or integral representations)
   (b) Taylor series and remainders for standard functions
   (c) Interchanging summation and integration
   (d) Example function which does not agree with its Taylor series (e.g., $f(x) = e^{-1/x^2}$)

8 Sample Texts

- Abbott, Stephen. Understanding Analysis, 2nd edition, Springer Undergraduate Texts in Mathematics, 2015.
- Bartle, Robert G. and Sherbert, Donald R. Introduction to Real Analysis John Wiley, 1982.
- Clark, Colin. Elementary Mathematical Analysis, 2nd edition, Wadsworth, 1982.
- Parzynski, William R. and Zipse, Philip W., Introduction to Mathematical Analysis, McGraw-Hill, 1982.

9 Bibliography

10 Prepared

February 16, 1985

11 Preparer

Prepared by K. Latil and M. Meck.

12 Modified

Modified by Owen Biesel and Aaron Clark, April 4, 2023.