MAT 372 Linear Algebra

Department of Mathematics
Southern Connecticut State University

I. Catalog Description

Course emphasizes matrices, systems of linear equations, vector spaces, elementary properties of linear transformation, eigenvalues, and applications.

II. Credit

(a) MAT 372 carries 3 semester hours of university credit.

(b) MAT 372 is required of all mathematics majors.

III. Prerequisite

C- or better in MAT 150.

IV. Format

MAT 372 is a lecture-based course.

V. Technology

Use of a computer algebra system (CAS) such as SageMath or Octave is strongly recommended.

VI. Course Objectives

The content of MAT 372 forms an important component of many areas of mathematics, both pure and applied. The course itself is a prerequisite for many of the more advanced mathematics courses. For these reasons, the course should be taken early in a student's undergraduate career, preferably concurrently with Calculus II (MAT 151). An important purpose of the course is to help prepare students for the rigor of MAT 373, MAT 480, or MAT 488. Thus, while the course itself is not a theoretical one, it does deal in an introductory way with some of the structure and abstraction inherent to its subject matter.

More specifically, by the end of the course, a successful student should be able to do the following:

(a) Find the general solution of a system of linear equations using row reduction

(b) Express a system of linear equations as a matrix equation.

(c) Find the determinant of a matrix by using cofactor expansion.

(d) Calculate the inverse of a matrix (if it exists) and use the inverse to solve a system of linear equations.

(e) Understand the definitions of subspace, basis, and dimension.

(f) Convert a spanning set for a subspace into a basis for the subspace.

(g) Determine if a vector lies in the span of a set of vectors.

(h) Determine if a set of vectors is linearly independent or dependent.

(i) Find the dimension of the span of a set of vectors.

(j) Find the kernel and range of a linear transformation.

(k) Compute the characteristic polynomial of a matrix.

(l) Compute all eigenvectors and eigenvalues of a matrix.

(m) Construct an orthonormal basis for a subspace using the Gram-Schmidt Process.

VII. Outline

Items in the outline marked with an asterisk indicate areas where technology will be integrated into the learning process. Students should be able to solve problems involving small matrices by hand. In later sections, when matrix algebra is required to solve problems related to linear systems or the properties of vector spaces and eigenspaces, students are encouraged to use a Computer Algebra System (CAS).

1. Matrix Algebra (25 %)

1.1. Matrices and Matrix Operations

(a) Definitions of matrices and special types of matrices (square matrix, n-vector, diagonal matrix, scalar matrix, identity matrix, and triangular matrix).

(b) Definitions and properties of matrix addition, scalar multiple, and linear combination of matrices.

(c) Definition and properties of transpose.

(d) Perform algebraic operations on matrices using a CAS.*

1.2. Matrix Multiplication

(a) Definition and properties of matrix multiplication. (Emphasis on the differences between matrix multiplication and ordinary multiplication of numbers.)

(b) Compute the product of matrices using a CAS.*

(c) Matrix-vector product, which can be written as a linear combination of column vectors of the matrix.

1.3. Reduced Row Echelon Forms

(a) Definition of row echelon form (REF) and reduced row echelon form (RREF).

(b) Elementary row operations and how to transform a matrix into its REF or RREF.

(c) Compute REF or RREF using a CAS.* (Students should be able to transform a simple matrix into its REF or RREF by hand, but for the remaining sections students will use calculators or CAS for the transformations.)

1.4. The Inverse of a Matrix

(a) Definition and properties of invertible matrices.

(b) How to determine a 2×2 matrix is invertible and to find its inverse.

(c) Find the inverse A^{-1} by transforming $[A \;\; I]$ into its RREF or directly using a CAS*.

(d) Application: Encoding and Decoding messages. Use a CAS to find the inverse matrix.*

1.5. Determinants

(a) Cofactor expansion to compute the determinant.

(b) Compute the determinant using a CAS.*

(c) Properties of determinants.

2. Linear Systems (15 %)

2.1. The Method of Elimination

(a) Definition of linear systems and their solutions.

(b) Gauss-Jordan Elimination. Use a CAS to find RREF* and solve a linear system. How to write the general solutions by using parameters.

(c) Determine whether a given linear system is consistent or inconsistent from its REF or RREF.

(d) Homogeneous linear systems and only trivial solutions.

2.2. Matrix Equations

(a) Rewrite a linear system as a matrix equation or as a vector equation.

(b) The relation between a matrix equation $A\mathbf{x} = \mathbf{b}$ and the vector $\mathbf{b}$ as a linear combination of the columns of A.

(c) The relation between an invertible matrix A and the matrix equation $A\mathbf{x} = \mathbf{b}$.

3. Euclidean Vector Spaces (25 %)

3.1. $\mathbb{R}^n$ as a Vector Space

(a) List of ten axioms that define $\mathbb{R}^n$ as a vector space.

(b) Definition and examples of subspaces of $\mathbb{R}^n$ including null spaces.

(c) Subspaces spanned by a set of vectors, and column spaces.

3.2. Linear Independence and Basis

(a) Definition of linear independence.

(b) Relation among linearly independent set, homogeneous matrix equation, and determinant.

(c) Definition of a basis. Examples of bases for subspaces including null spaces and column spaces.

3.3. The Dimension of a Vector Space

(a) Definition and examples of the dimension.

(b) Basis Theorem.

(c) Definitions of rank and nullity and the Rank Theorem.

4. General Vector Spaces (20 %)

4.1. Real Vector Spaces and Subspaces

(a) Definition and examples of real vector spaces. (Matrices, Polynomials, and Functions)

(b) Prove basic properties of vector spaces using the axioms.

(c) Definition and examples of subspaces. Spanning sets.

(d) Linear independence, basis, and dimension.

(e) Ordered basis and coordinate vectors.

4.2. Linear Transformations

(a) Definition and examples of linear transformations.

(b) Properties of linear transformations.

(c) Kernel and range. Nullity and rank of a linear transformation.

(d) Definition and examples of coordinate mappings.

(e) Use coordinate mappings to determine the spanning, linearly independence, and basis.

5. Applications (15 %)

5.1. Eigenvalues and Eigenvectors

(a) Definitions and examples of eigenvalues, eigenvectors, and eigenspaces.

(b) Characteristic polynomials and equations*.

(c) By using the given eigenvalues, find the bases for the eigenspaces*.

(d) Diagonalization*.

5.2. Orthogonality

(a) Inner product and length. Orthogonal and orthonormal bases.

(b) The Gram-Schmidt Process*.

(c) Least-Squares*.

VIII. Topics for Projects

The following topics are suggested for projects that utilize a computer algebra system, such as SageMath. We recommend assigning these topics independently, rather than necessarily covering them in class.

(a) Applications of linear systems: Balancing the chemical equations, network analysis, and electrical network.

(b) Applications of matrices: Graphs and digraphs, Computer graphics, and Fractals.

(c) Applications of eigenvalues: Systems of differential equations, and Markov Chains.

(d) Applications of orthogonality: Mathematical Modeling Using Least Sqaures.

IX. Assessment

Individual instructors may vary assessment modes, but typically grades will be based on in-class assignments, quizzes, tests, and problem sets.

X. References

(a) S. Lay, J. McDonald, and D. Lay, *Linear Algebra and its Applications*, 6th edition, Pearson.

(b) H. Anton and C. Rorres, *Elementary Linear Algebra*, 11th edition, Wiley.

(c) L. Spence, A. Insel, and S.H. Friedberg, *Linear Algebra*, 5th edition, Pearson.

(d) R. Larson, *Elementary Linear Algebra*, 8th edition, Cengage.

XI. Waiver Policy

This course may not be waived.

XII. Preparation

- Revised by J. Hong and L. Sturman, February 2025.
- Approved by the MDCC on February 25, 2025.
- Approved by the Math Department on April 24, 2025.