Southern Connecticut State University
MAT 322–Numerical Analysis I

I. Description

(A) **Catalog Description**: Topics include approximate solutions of equations, polynomial approximations of functions, interpolation, numerical differentiation and integration, and their corresponding techniques of error analysis. A graphing calculator approved by the instructor is required. Computer programming is also required.

(B) **Expanded Description**: Students will be exposed to both the mathematical and practical aspects of numerical methods, including technical programming. The course begins with a discussion of the differences between exact arithmetic and computer arithmetic, but quickly transitions into the development and analysis of algorithms for root finding, interpolation, and numerical calculus. Particular attention is paid to error analysis for the various methods. The algorithms, including derivations where appropriate, take front stage, but this comprises perhaps half the content of the course. The other half is divided approximately equally between error analysis and programming. The hope is to engrain the philosophy that "a numerical method without any way to predict its accuracy is not a numerical method." Taylor Polynomials and their remainder terms are visited extensively throughout the course.

II. Credit

(A) MAT 322 carries four (4) semester hours of college credit.

(B) MAT 322 is required of all mathematics majors in the B.S. in Applied Mathematics degree program.

(C) MAT 322 does not satisfy the All-University Requirement in mathematics.

III. Prerequisite

(A) The prerequisite courses are MAT 151 (Calculus II) and CSC 152 (Computer Programming I). Specifically, the following material is prerequisite:

1. Differential and Integral Calculus, including Taylor Polynomials.
2. Programming competence.

(B) No placement test or skills test is necessary.

IV. Format

(A) MAT 322 is primarily a lecture-based course.

(B) A graphing calculator is required. A programmable calculator is required if the programming component of the course is to be done on the calculator.[1]

(C) Programming is required.[1]

(D) Use of a computer algebra system (CAS) is optional.[1]

V. Outline

In addition to the mathematical topics outlined, it is expected that students will write computer programs that implement most, if not all, methods using basic programming skills.[1] Additionally, students must demonstrate competent independent programming skills by writing several computer programs that implement algorithms/methods for which no pseudo-code nor sample code is provided. These algorithms/methods may be extensions or modifications or additions to the basic algorithms whose pseudo-code or sample code is given.

[1] The programming component of the course is to be done without the use of built-in numerical procedures or symbolic capabilities of the calculator or computer algebra system.

(A) Preliminaries (15%)

1. Taylor polynomials and remainder theorem
2. Computer arithmetic, including round-off errors and the limits of computer precision
3. Big O notation

(B) Approximate solutions of equations (30%)

1. The Bisection Method and error bounds
2. Fixed-point Iteration
   - existence and uniqueness theorems on fixed point convergence
   - error bounds
   - Newton's Method and the Secant Method
   - order of convergence
   - acceleration of convergence (Aitken's Δ^2 and Steffensen's methods)
3. Zeros of polynomials
   - Synthetic division
   - Using Horner's Method to implement Newton's Method
   - Deflation

(C) Polynomial Approximation and Interpolation (30%)

1. Lagrange polynomials and error bounds
2. Divided differences
3. Osculating polynomials and Hermite interpolation
   - error bounds
   - tabular computation
4. Cubic spline interpolation
5. Bézier curves
   - Computation of the defining cubic functions
   - Interactive manipulation using appropriate software

(D) Numerical Differentiation and Integration (25%)

1. Derivative approximation
   - error bounds
   - derivation of formulas via Lagrange polynomials
   - derivation of formulas via Taylor polynomials
2. Richardson's extrapolation
3. Definite integral approximation
   - error bounds
   - derivation of formulas via Lagrange polynomials
   - derivation of formulas via Taylor polynomials
   - degree of accuracy
   - composite integration
   - adaptive quadrature

VI. Proposed Text

Burden, R., and Faires, J., *Numerical Analysis,* eighth edition, Thomson Brooks/Cole, 2005.

Recommended sections (about 19 sections):

- Chapter 1: 1.1 (focus on Taylor polynomials), 1.2 (all), 1.3 (Big O notation)

- Chapter 2: 2.1-2.5 (all), 2.6 (Horner's/Newton's method)
- Chapter 3: 3.1-3.5 (all)
- Chapter 4: 4.1-4.4, 4.6 (all)

VII. Bibliography

Atkinson, K., *Elementary Numerical Analysis*, Wiley, 1985.

Bradie, B., *A Friendly Introduction to Numerical Analysis*, Pearson Prentice Hall, 2006.

Burden, R. and Faires, J., *Numerical Analysis*, eighth edition, Thomson Brooks/Cole, 2005.

Conte, S. and de Boor, C., *Elementary Numerical Analysis: An Algorithmic Approach*, McGraw-Hill, 1965.

VIII. Goals

(A) Achieve mastery of a rich and diverse set of mathematical ideas.

(B) Use acquired mathematical skills to undertake independent learning and to be a contributing member of a problem solving team.

(C) Demonstrate the ability to use and understand multiple representations (including graphical, numerical and analytical) of mathematical concepts.

(D) Understand and appreciate connections among different areas of mathematics and with other disciplines.

(E) Utilize appropriate technology to develop models for solving problems and analyzing new situations.

(F) Appreciate the beauty, joy, and challenge in mathematics and experience mathematics as an engaging field with contemporary open questions.

(G) Demonstrate competency in a programming language.

(H) Demonstrate a command of ideas and techniques ranging across single and multivariate calculus, discrete mathematics, linear algebra, probability and statistics, and differential equations.

IX. Outcomes

Students passing MAT 322 should be able to do each of the following tasks. Tasks (A)-(F) should be done by hand (with the assistance of basic, non-symbolic, calculator functionality where appropriate).

(A) Compute Taylor polynomials to modest degree.

(B) Find theoretical bounds on the error of every approximation for which a suitable theoretical error bound exists.

(C) Demonstrate the computation of every method discussed for small numbers of steps or iterations.

(D) Be able to compare and contrast the relative strengths and weaknesses of the various root finding methods.

(E) Be able to compare and contrast the relative strengths and weaknesses of the various interpolation methods.

(F) Derive via Lagrange polynomials or Taylor polynomials various methods of approximating derivatives and definite integrals.

(G) Program, using only basic programming functions, an implementation of any method given pseudo-code.

(H) Use programs to complete computations that would be too lengthy to complete with a pencil, paper, and (unprogrammed) calculator.

(I) Program, using only basic programming functions, an implementation of methods for which no pseudo-code nor sample code is given.

X. Waiver Policy

There is no waiver policy for MAT 322.

XI. Preparation and Approval

Prepared on 16 May 2006.

Approved by the MDCC on .

Approved by the department on .

XII. Preparers

Prepared by Leon Brin and Therese Bennett.