T2AE 298/ MAT 210   Analysis of Election Structures

Department of Mathematics
Southern Connecticut State University

I.   Catalog Description

Students will explore the structural consequences of democratic processes of elections. Discussions and activities will center around historical and hypothetical scenarios that weigh the political and social ramifications of a variety of election structures. Topics include representation systems, voting formats, and gerrymandered district design.

II.   Credit

- MAT 210 carries 3 semester-hours of university credit.

- MAT 210 satisfies the University's Liberal Education Program American Experience requirement.

III.   Prerequisites

Quantitative Reasoning and Technological Fluency co-requisite

IV.   Technology

Students will need to be able to perform basic function calculations within a spreadsheet.

V.   Liberal Education Program Requirements

This course fulfills the Exploration American category of our general education requirements, the Liberal Education Program. The purpose of the American Experience category is to develop a broad understanding of the society, politics, and culture of the United States and in particular its historical and contemporary diversity. This knowledge enables students to become informed and engaged citizens and provides a social and historical context to their lives.

    American experience courses should contain exposure to all of the below with emphasis on at least one.

1. (AE - 1) Historical Development: Examining the evolution of American society, culture, and/or politics and interconnections among them.

2. (AE - 2) Diversity in American Experience: Understanding the roles of many different peoples in creating the American experience including non-majority perspectives.

3. (AE - 3) America's Relationship with the World: Exploring historical and recent issues concerning America's interactions with the world (e.g., immigration, globalization, etc.).

In this course, we cover these objectives and we will also reemphasize the following Foundations competencies: Critical Thinking, Quantitative Reasoning, Technological Fluency, and Written Communication. Finally we cover the "Discussion of Values" in the area of Civic Engagement.

How this course will meet each of the above LEP requirements:

1. - The course will deeply expose students to Historical Development (**AE-1**):

   (a) Within the first unit on apportionment, students will delve into the creation of the electoral college from its founding within the debates at the Continental Congress as they debated the impacts of Jefferson and Hamiltonian methods. Then students will explore how the calculation of House of Representatives state apportionment has changed over the years to the current Huntington-Hill formula and explore how various methods would have impacted presidential (and other) vote outcomes throughout history.

   (b) Within the unit on voting methods, we will explore various results from varied voting methods used in different elections, many of which have been enacted for localized political and/or societal issues (e.g. the adaptation of voting procedures in underrepresented communities both to silence such voices or to elevate this population's voice).

   (c) Similar to the discussion from the use of voting methods, the unit on district design will explore historical and contemporary examples of legal and illegal districting to emphasize or de-emphasize the impact of a community's vote and representation.

2. The course will expose students to the Diversity in American Experience (**AE-2**) in the unit on District design, as often gerrymandering and voter suppression shift along with changing population demographics.

3. The course will expose students to America's Relationship with the World (**AE-3**) through investigating the effects of apportionment and district design in states where the population has undergone major shifts due to immigration.

4. Throughout the course, students will be asked to calculate apportioned representation, voting results, voting power indices, and other coefficients surrounding democracy representation using spreadsheet functionalities, and to digitally report the results. Also, during the projects, students will be asked to critically consider various scenarios for how they affect democratic outcomes and provide written reflections on their findings. All units culminate in reemphasizing the foundational skills of Critical Thinking (**CT**), Quantitative Reasoning (**QR**), Technological Fluency (**TF**), and Written Communication (**WC**).

5. The content of the course provides the students ample opportunity to reflect and analyze their role in the process of civic engagement (**CE**) and the effects their voice and participation have on our local, regional, and national democratic process.

6. Students in this course will be asked to read current expository articles related to the subject such as "Geometry versus Gerrymandering", "Designing Stable Elections", or "The Malapportionment of the US House of Representatives: 1940–2020" to compare claims made there with the knowledge they have gained in the course.

VI.   Course Objectives

In addition to satisfying requirements, MAT 210 has some specific course objectives. By the end of the course, a successful student should be able to do the following:

1. Identify and weigh the pros and cons of different methods of apportioning representation.

2. Understand the various desirable properties of different voting systems, as well as their trade-offs.

3. Evaluate district maps through diverse notions of fairness and reasonableness.

4. Use relevant historical examples to enrich discussion of apportionment, voting methods, and district design.

5. Make predictions about the social justice consequences of structural changes to democratic processes.

VII.   Outline

Apportionment ($\sim 40\%$):

(a) Proportional Representation - students will explore the idea of representation with fictional examples to establish what seems "fair" - e.g. How many representatives should each sports team get on an athletic council? or How many faculty representatives should each department get at a curricular council?

(b) Compare and contrast different U.S. House of Representative state apportionment methods. - ex. Jefferson, Hamilton, Webster, and Huntington Hill Methods.

(c) Explore Paradoxical scenarios within different apportionment methods - ex. New States Paradox, Alabama Paradox, etc.

(d) Analyze the effects of apportionment methods on Electoral College and historical Presidential Election results

(e) Investigate Voter Power Indices - e.g. Within the electoral college voting system, who has the most power? While a voter in a low populated state like Wyoming have a high ratio of voter per electorate, it is unlikely that the outcome of Wyoming's vote would actually change the Presidential vote winner. On the contrary, highly populated states like California have a low ratio of voter per electorate. However, if it happened to be that California was deadlocked when the last voter entered the polling booth, this person's vote would likely decide the President elect. So how do we calculate where the power lies within a voting system?

Voting Methods ($\sim 30\%$):

(a) Exploration of Voting Methods - Students will work with fictional and historical voting data to explore various voting methods. Methods should include but are not limited to the following:

- Plurality Methods (runoff, elimination, etc.)
- Borda Count Methods
- Weighted Majority
- Approval

(b) Strategic Voting - e.g students will explore how a voter can strategically cast votes to have the greatest desired election outcome, even when the vote they cast does not match their specific ideals/desires.

(c) Fairness Criteria - students will examine voting methods as meeting the standard fairness criteria. The course should cover at least the following criteria

   i. Majority

   ii. Head-to-Head

   iii. Monotonicity

   iv. Independence of Irrelevant Alternatives

(d) Impossibility Theorems - after exploring numerous methods under all fairness criteria, the class will discuss how no voting method can meet all fairness criteria. (Students will not be expected to understand the proofs of these theorems.)

   i. Arrow's

   ii. Gibbard's

District Design ($\sim 30\%$):

(a) Rectilinear Gerrymandering - students will explore the idea of districting by working in fictional "grid cities" where the voters from a square in the grid must belong to the same district. This will expose students to the following real life concepts:

- Census Blocks
- Population/Error Coefficients
- State Constitutional and/or Geographical restraints.
- Fairness Criteria - before and after census/voting totals

(b) Voter Suppression - students will look at the history of Gerrymandering and Polling accessibility from the lens of population/voter turnout.

(c) Measures of Gerrymandering - Students will look at sample state districts under the lens of the following known metrics for identifying gerrymandered districts.

- Efficiency Gap
- Compactness
- etc.

(d) Historical Court Rulings - students will look at the recent history of court rulings on legal and illegal districting and how the mathematical process of randomization was used in these cases.

Example Weekly Schedule

Week	Content
1	1. Introduction to apportionment via IBL activity - "Class Trip to Europe" - on deciding fair partition based on voting. 2. Overview of Constitutional Convention debates in regards to apportionment (e.g. HoR, Article 2 of U.S. Constitution, 3/5ths Compromise, etc.) 3. Contextual concept of who is being represented (e.g white land owners) compared with today's understanding of representation.
2	1. Compare and Contrast Hamilton vs Jefferson's Methods of Apportionment considered in 1792. 2. Context for the selection of Jefferson's method over a previously vetoed Hamilton's method.
3	1. Affects of the 12th amendment on the presidential election process. 2. Look at Paradoxical scenarios (Alabama, New States Paradox) in both Jefferson and Hamiltonian Methods. 3. A Case Study of alternate apportionment - Webster's Method (1842) - affects on the 1844 and 1848 elections.
4	1. Hamilton's Method adopted (1850) and reconstruction amendments (and laws) and their affects on apportionment. 2. Additions of New States (Case Studies such as Oklahoma) affecting apportionment. 3. Re-examine representation of peoples under U.S. law. (e.g. 15th and 19th amendments, indigenous persons, and populations of territories.)
5	1. 3rd Party Candidacy Case Studies- 1892 or 1912 Presidential Election. 2. Huntington-Hill Method adopted (1929/1940) 3. Modern Elections Case Studies - 1992 3rd Party Candidacy, 2000/2016 popular vote vs Electoral College winner
6	1. Exploration of Voter Power Indices. 2. Voting behaviors and turn out throughout history. 3. Project 1 Assignment
7	1. Introduction to Various Voting Methods
8	1. Case studies on voting methods in local and state elections 2. Explore voting methods in use outside U.S. 3. Defining fairness criteria

9	1. Analyze voting procedures against fairness criteria
	2. Strategic voting methods
	3. Exposure to Arrow's Impossibility Theorem
10	1. Exposure voting theorems (Arrow and Gibbard)
	2. Project 2
11	1. Introduction to Districting via Rectilinear Gerrymanding activity
	2. Origination of term Gerrymandering (Boston 1812)
	3. Case studies of early re-districting efforts to retain political power.
12	1. Why the need for districting
	2. Defining districting variables - census blocks, geographical constraints, state laws/terminology (e.g. compactness)
13	1. Defining methods of manipulative districting (e.g. packing, cracking, hijacking, etc.)
	2. Voter suppression via polling, districting, and accessibility
14	1. Measures of districting
	2. Voter suppression via polling, districting, and accessibility
15	1. Case Studies of Gerrymandered districts - proven and unproven
	2. Project 3

VIII.   Assessment

Individual instructors may vary assessment modes, but typically grades will be based on a combination of class activities, quizzes, and projects. The expectation is that the bulk of a students grade will come from non high stakes assessment, as many concepts and methods in this course will require the use of research and spreadsheets to understand and analyze. The memorization of all the nuances is beyond the expectation of a 200 level general education course.

Example Course Grade Calculation

- Class Attendance and Participation - 10% total

- 15 Group Class Activities - 25% total

- 10 Quizzes - 20% total

- 3 Unit Project - 15% each (45% total)

IX.   Bibliography

(A) Barnett, Janet Heine. "The French Connection: Borda, Condorcet and the Mathematics of Voting Theory," Convergence (September 2020).

(B) Duchin, Moon "Geometry versus Gerrymandering." Scientific American, Nov. 2018. `https://www.scientificamerican.com/article/geometry-versus-gerrymandering/`

(C) Ellenberg, Jordan S. "Geometry, Inference, Complexity, and Democracy." Bulletin of the American Mathematical Society 58, no. 1 (November 2, 2020): 57–77. `https://doi.org/10.1090/bull/1708`

(D) Heilman, Steven. "Designing Stable Elections" Notices of the American Mathematical Society 68, no. 4 (April 2021):516-527. `https://www.ams.org/notices/202104/rnoti-p516.pdf`

(E) Li, R. (2022). "The Malapportionment of the US House of Representatives: 1940–2020." PS: Political Science & Politics, 55(4), 647-654.

(F) Stephanopoulos, N., and McGhee, E. "Partisan gerrymandering and the efficiency gap." U. Chi. L. Rev. 82: 831 (2015)

(G) Wallis, Walter Denis. "The mathematics of elections and voting." Heidelberg: Springer, 2014.

X. Waiver Policy

MAT 210 may not be waived.

XI. Preparation

- Proposed outline prepared by Andy Bartlett, Owen Biesel, and Braxton Carrigan, September, 2023.

- Approved by the MDCC, 8-0-1 September 13, 2023.

- Approved by the Mathematics Department, 16-0-0 September 27, 2023.