MAT 115 Introduction to Modern Mathematics

Department of Mathematics

Southern Connecticut State University

I. Catalog Description

A collection of introductions to, and surveys of, several active and important topics in mathematics. Topics include cryptography, data science, statistics, actuarial sciences, fractals, graph theory, discrete geometry, topology, game theory. Prerequisite: None. 3 credits.

II. Purpose

Many students pursuing a mathematics degree have only been exposed to high school algebra and calculus. MAT 115 will expose students, early on in their time at Southern, to how mathematics is being used in business, industry, and government, as well as what mathematics is being studied by researchers. The format of the course will be similar to a seminar course, where for each topic, the instructor will spend a day presenting background on the topic being covered. A faculty member will then give an overview of what the topic is and how it is being used/researched. Students will then spend the next class working on solving problems in this topic. This course is intended to be taken by students during the first year as a mathematics major. At the end of the course, students will be more prepared for the mathematics they will see after the calculus sequence.

III. Credit

MAT 115 carries three semester-hours of university credit. It is required of mathematics majors.

IV. Prerequisites

MAT 112 corequisite

V. Format

MAT 115 meets for 3 contact hours per week throughout a standard academic semester and is conducted primarily in a lecture and discussion format; however, discovery-based learning and group work are highly encouraged.

VI. Course Objectives

Upon completion of MAT 115, students should:

A. Understand mathematics is not just algebra and calculus.

B. Understand how mathematics is being used in business, industry, and government.

C. Make connections between the mathematics they have studied so far and the mathematics they will be studying in their future courses.

D. Learn new techniques to solve mathematics problems, outside of using their algebra skills.

E. Be able to use relevant technology for the topics covered in the course.

VII. **Outline**: Choose $5-7$ of the following topics, spending $2-3$ weeks or so on each topic.

A. Cryptography
   1. Encryption and Decryption of a Message
   2. Methods used in Cryptography
   3. Mathematics of Cryptocurrency

B. Data Science
   1. Machine Learning
   2. Artificial Intelligence

C. Actuarial Sciences
   1. Markov Chains
   2. Financial Computational Models
   3. Future Lifetime Random Variables and Life Tables

D. Fractals
   1. Julia and Mandelbrot Sets
   2. Iterative Function Systems
   3. Random Walks and Brownian Motion

E. Graph Theory
   1. Models for Optimization Problems in Road Networks
   2. Molecular Epidemiology
   3. Graph Colorings and Planarity
   4. Random and Extreme Graphs

F. Computational Geometry
   1. Art Gallery Problem
   2. Linkages
   3. Origami

G. Topology
   1. Knot, Links and Braids
   2. Classification of Surfaces

H. Countability and Cardinality
   1. Axiom of choice
   2. Continuum Hypothesis
   3. Different Infinities

I. Game Theory
   1. Two-person zero and non-zero sum games
   2. Two-person non-zero sum games
   3. Economic game theory.
   4. Combinatorial Game Theory
   5. Four-Color Theorem

J. Voting Theory
   1. Voting Methods
   2. Voting Theory and Fairness
   3. Gerrymandering

K. Design Theory
   1. Latin Square Construction
   2. Steiner Triple System
   3. Block Design

L. Root Finding Methods
   1. Fixed Point Iteration
   2. Newton's Method
   3. Bisection Method

M. Optimization
   1. Linear Programming
   2. Random Grid Search
   3. Particle Swarm

N. Unsolved and Recently Solved Conjectures
   1. Riemann Hypothesis
   2. Poincare Conjecture
   3. Hilbert's List
   4. Goldbach's Conjecture
   5. Twin Prime Conjecture
   6. $3x + 1$ Conjecture
   7. P vs NP Conjecture
   8. Erdős Primitive Set Conjecture

VIII. Assessment

Individual instructors may vary assessment modes, but typically grades will be based on in-class assignments, problem sets, mini-projects, and a presentation.

IX. Recommended Texts

None

X. Waiver Policy

This course may be waived.

Bibliography

A. Kenneth Rosen, *Elementary Number Theory and Its Application*, 6th edition, Pearson, 2010.

B. Peter Wentworth, Jeffrey Elkner, Allen B. Downey, and Chris Meyers, *How to Think Like a Computer Scientist: Learning with Python*, 3rd Edition, 2012.

C. Dale S. Borowiak and Arnold F. Shapiro, *Financial and Actuarial Statistics (An Introduction)*, Second Edition, CRC Press, 2014.

D. K. J. Falconer, *Fractals: A Very Short Introduction*, OUP Oxford, 2013.

E. Reinhard Diestel, Graph Theory, Springer, 5th Edition, 2017

F. Mark de Berg, Otfried Cheong, Marc van Kreveld, and Mark Overmars, *Computational Geometry: Algorithms and Applications*, Springer, 3rd Edition, 2008.

G. V.V. Prasolov, *Intuitive Topology*, AMS, 1994.

H. Joseph Fields, *A Gentle Introduction to the Art of Mathematics*, Version 3.1, `https://giam.southernct.edu/GIAM/GIAM.pdf`

I. Steven Tadelis, *Game Theory: An Introduction*, Princeton University Press, 1st Edition, 2013.

J. Jonathan K. Hodge and Richard E. Klima, *The Mathematics of Voting and Elections: A Hands-On Approach*, MAA, Second Edition, 2018.

K. Charles C. Lindner and Christopher A. Rodger, *Design Theory*, CRC Press, 2nd Edition, 2017.

L. Richard L. Burden, J. Douglas Faires, and Annette M. Burden, *Numerical Analysis*, Cengage, 10th Edition, 2015.

M. Edwin K. P. Chong and Stanislaw H. Zak, *An Introduction to Optimization*, Wiley, 3rd Edition, 2013.

N. Ben Yandell, *The Honors Class: Hilbert's Problems and Their Solvers*, CRC Press, 1st Edition, 2001.

O. Keith Devlin, *The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles Of Our Time*, Basic Books, 2003.

XI. **Preparation**

Proposed outline prepared by K. Kruczek and A. Clark, Fall 2021.

Approved by the MDCC, September 27, 2022.

Approved by the Mathematics Department, September 10, 2030