Southern Connecticut State University School of Arts and Sciences Department of Mathematics

OUTLINE FOR MAT 106 Mathematics for Elementary Education II

I. Catalog Description.

Topics include geometry, measurement, rational numbers, irrational numbers, ratio, proportion, percent, problem solving, mathematical reasoning and connections, probability and statistics.

II. Purpose.

The purpose of this course is to continue to provide students with a conceptual framework and deeper understanding of the theory behind the mathematics being taught in elementary schools. It is required of all students in an elementary school certification program.

III. Credit. MAT 106 carries 3 semester hours of university credit.

- **IV. Quantitative Reasoning.** This course does satisfy the University's Liberal Education Program (LEP) requirement in Quantitative Reasoning (QR). It addresses the key elements of QR as indicated in section VIII, Course Objectives. Further, as a Tier 1 LEP course, it will
- address at least one *Area of Knowledge and Experience*, e.g. Cultural Expressions (tangram activity or geometry or symmetry), Global Awareness (Pythagorean Theorem or geometry); or Natural World (statistics or measurement); Mind and Body (percents, proportions or statistics);
- incorporate at least one *Discussion of Values*, e.g. Human Diversity (tangrams, statistics), Rational Thought (geometric constructions or spatial visualization), Civic Engagement (using statistics to make a case for change), or Ethical Judgment (discussion re. emphasis on drill/skill practice vs. problem-solving activities into their classroom -- the "Math Wars" debate);
- address at least one embedded competency in a significant manner, e.g. Creative Thinking (project/poster), Information Literacy (library project), or Oral Communication (class presentations), and
- present Quantitative Reasoning in context.

V. Prerequisites.

C- or better in Math 105.

VI. Format.

MAT 106 is offered in the lecture-discussion format with a laboratory component. This course will have four contact hours per week and carry a faculty teaching load of 3.5 credits.

VII. Technology.

A graphing calculator is recommended.

VIII. Course Objectives.

- 1. Students will develop and extend their understanding of fractions, decimals, ratio and proportion, and percents. (NCATE 9.2, 9.4, INTASC 1, CCCT 1.3, QR1, QR3, QR4, QR5)
- 2. Students will recognize and generate equivalent forms of commonly used fractions, decimals, and percents, and be able to compare and order these numbers. (NCATE 9.2, 9.4, INTASC 1, CCCT 1.3)
- 3. Students will understand, and be able to model, arithmetic operations on fractions and decimals, and will be able to estimate the results of arithmetic operations on fractions and decimals. (NCATE 9.3, 9.4, INTASC 1, CCCT 1.3, OR1, OR3, OR4, OR5)
- 4. Students will understand and apply the basic concepts of probability, odds, and mathematical expectation. (NCATE 12.3, INTASC 1, CCCT 1.3)

- 5. Students will be able to collect data, represent the data in various forms, and draw conclusions from the data. (NCATE 12.1, 12.2, INTASC 1, CCCT 1.3, QR2)
- 6. Students will be able to compute measures of central tendency (mean, median, mode) measures of variation (range, variance, standard deviation) and understand what each measure indicates for a given set of data. (NCATE 12.2, INTASC 1, CCCT 1.3, QR4)
- 7. Students will be able to classify shapes according to their properties. (NCATE 11.1, INTASC 1, CCCT 1.3)
- 8. Students will be able to compute perimeter, area, and volume of basic 2- and 3-dimensional geometric shapes; measure angles; and understand the relationships among these measures for similar shapes. (NCATE 13.1, 13.2, INTASC 1, CCCT 1.3, QR4)
- 9. Students will understand the metric system and be able to perform computations in the system. (NCATE 13.2, INTASC 1, CCCT 1.3)
- 10. Students will communicate and be assessed on their thinking in a variety of methods: group work (Dept. 2), individual tests, direct instruction and other varieties (NCATE/NCTM 3.1-4, 6.1, 8.1; CCCT I.4, II.5, II.7) such as, teaching and planning mini-lessons (NCATE/NCTM 8.4, 8.7-9; CCCT I.5-6), graphing calculators (NCATE/NCTM 7.5-6).

IX. Laboratory.

The Mathematics Department maintains a laboratory and laboratory materials for MAT 106 in the Eureka Center, Engleman D125. Normally, all sections of MAT 106 are scheduled in the Eureka Center. Laboratory activities are integrated into the course and may include, but are not limited to, the following activities: tangram, Cuisinaire rods, pattern blocks, ratio lab, base ten blocks, probability sampling, lab activity to collect and present data, geometric constructions, origami, Mira, Geoboard, Platonic Solids, area/perimeter relationship lab, and volumes of pyramids vs prisms lab.

Though these labs are all hands-on activities, one is welcome to include technology labs as one sees fit. For example, fractions and decimal operations are topics we expect elementary teachers to do without calculators, but probability simulations are good to do on the calculator (the TI has a random number generator and has a program that rolls dice, deals cards and spins a spinner.). There are also Calculator-Based Laboratory and Calculator-Based Ranger (CBL/CBR) available for use in these classes. There are many opportunities in this class to highlight new innovations and still focus on the concrete understanding a child should have before learning in more abstract settings.

X. Outline.

Percentages are based on a 28 class semester, with 5 classes reserved for testing and review. The chapter sections are from the recommended text below. The starred * sections are optional and the ** sections are also optional and can be used for group projects, if time permits.

- A. Rational numbers 17% (Chapter 6)
 - 1. The Set of Rational Numbers
 - a. Models and interpretations
 - b. Equivalent fractions
 - c. Order and density of fractions
 - 2. Addition and Subtraction of Rational Numbers
 - 3. Multiplication and Division of Rational Numbers
- B. Decimals and Real Numbers 13% (Chapter 7)
 - 1. Introduction to Decimals
 - a. Models and interpretations
 - b. Equivalent representations including terminating and infinite repeating
 - c. Order of decimals
 - 2. Operations on Decimals
 - a. Addition and Subtraction
 - b. Multiplication and Division

- 3. Nonterminating Decimals
 - a. Irrationality of the square root of 2
 - b. Infinite nonrepeating decimals
 - c. Order
 - d. Real Numbers (include Pythagorean Theorem and roots)
- C. Ratio and Proportion 10% (Chapter 8)
 - 1. Proportional Reasoning
 - 2. Percents
 - 3. Computing Interest *
- D. Probability 17% (Chapter 9)
 - 1. How Probabilities Are Determined
 - a. Theoretical vs. experimental probabilities
 - b. Law of Large Numbers
 - c. Single stage experiments
 - 2. Multistage Experiments with Tree Diagrams and Geometric Probabilities
 - 3. Using Simulations in Probability
 - 4. Odds, Conditional Probability and Expected Value
 - 5. Methods of Counting *
- E. Statistics 13% (Chapter 10)
 - 1. Statistical Graphs
 - 2. Measures of Central Tendency and Variation
 - a. Mean, median, mode
 - b. Box-and-whisker plots
 - c. Variance and standard deviation
 - 3. Abuses of Statistics
 - a. Simpson's Paradox (Not in current text. Go to http://www.askdrmath.com and search for Simpson's Paradox, use the Wikipedia reference http://en.wikipedia.org/wiki/Simpson's_paradox or search the web for Simpson's paradox or Stein's paradox)
 - b. Other Abuses of statistics and statistical graphs
- F. Geometry 17% (Chapter 11)
 - 1. Basic Notions
 - a. Point, line, and plane
 - b. Angles and properties
 - 2. Polygons
 - 3. More About Angles
 - 4. Geometry in Three Dimensions
 - 5. Networks *
 - 6. Congruence Through Constructions ** (can use interactive geometry software, origami, or manipulatives such as Sketchpad, Miras, or compass/straightedge) (12-1)
 - 7. Other Congruence Properties * (12-2)
- G. Measurement 13% (Chapter 13)
 - 1. Reliability of Data (representation of accuracy in measurements: not in current text, use supplement presented in item XIII.)
 - 2. Linear Measure
 - 3. Areas of Polygons and Circles
 - 4. The Pythagorean Theorem
 - 5. Surface Areas **
 - 6. Volume, Mass, and Temperature **
- H. **Symmetry (Chapter 14)

- 1. Translations and Rotations
- 2. Reflections and Glide Reflections
- 3. Size Transformations
- 4. Symmetries

XI. Texts.

Billstein, R., Libeskind, S. & Lott, J. *A Problem Solving Approach to Mathematics for Elementary School Teachers*, 10th Ed., Pearson Addison-Wesley, 2010.

Supplement on representation of data (section XIII)

XII. Waiver Policy.

There is no waiver for MAT 106.

XIII. Supplement on representation of data.

The decimal representation of numbers can be extended to communicate an additional bit of information: If the number represents a measurement, there is a question about the accuracy of the measurement.

For instance, you may say that you are 63 inches tall (5 feet, three inches tall), but it is likely that you are not EXACTLY that tall. In fact, you know that you can change your height by standing taller or by measuring in the evening after standing all day. Even two people measuring you simultaneously with a very accurate tape might give different measurements just because of the way they look at the tape. Still, when you say that you are 5 feet 3 inches tall, people have a pretty good idea how tall you are.

When giving a measurement, you can quickly and easily communicate its accuracy by using the correct number of decimal places in the number: just round to the appropriate decimal place. For instance, if you are 63 inches tall to the nearest inch, report it as "63. inches tall." If you are 63 inches tall to the nearest tenth of an inch, report it as "63.0 inches tall" and so on.

COURSE CONTRIBUTION:

MAT 106 is the second semester in the MAT 105-106 sequence for elementary education majors. It is mainly a content course, but methodology is discussed as well. This course has a laboratory component built into it, thus students meet for four (4) hours per week. The laboratory is used primarily for hands-on activities.

The mathematics content is divided into three main areas: 1) extension of number systems, 2) probability and statistics, and 3) geometry and measurement.

Having studied the whole numbers, natural numbers, and integers in MAT 105, students develop an understanding the rationals, irrationals, and reals in MAT 106. Students develop an understanding of multiple representations of these numbers, including decimals (terminating, repeating, non-terminating/non-repeating, fractions, and percents), as well as models for the four arithmetic operations on these numbers. Estimation skills are stressed throughout. Students use manipulatives to develop an understanding of algorithms on rational numbers.

Students develop an understanding of concepts of probability, expected value, and odds, through the use of simulations. Students explore statistical concepts through data collection, representing data in multiple forms, and drawing conclusions from their data.

Students develop an understanding of properties of shapes, including perimeter and area. Students learn to use various scales of measurement, including the metric system.

STANDARDS GUIDELINES

INTASC STANDARDS

[Interstate New Teachers' Assessment & Support Consortium]

S

- 1. Knowledge of subject matter
- 2. Knowledge of human development & learning
- 3. Instruction adapted to meet diverse learners
- 4. Use of multiple instructional strategies & resources

A

- 5. Effective learning environment created
- 6. Effective communication
- 7. Lesson planning

I

9. Reflection and professional development

L

8. Assessment of student learning to improve teaching

S

10. Partnership with school and community

PROFESSIONAL STANDARDS

NCATE - NCTM STANDARDS

[National Council of Teachers of Mathematics]

OUTCOMES FOR GRADES K-4 TEACHERS WITH MATHEMATICS EMPHASIS

Programs prepare prospective teachers who can –

Communicate their mathematical thinking orally and in writing to peers, faculty, and others (3.1-3.4)

Recognize, use, and make connections between and among mathematical ideas and in contexts outside mathematics to build mathematical understanding (4.1-4.3)

Possess a deep understanding of how students learn mathematics and of the pedagogical knowledge specific to mathematics teaching and learning (8.1)

Demonstrate computational proficiency, including a conceptual understanding of numbers, ways of representing number, relationships among number and number systems, and the meanings of operations (9.2-9.4)

Use spatial visualization and geometric modeling to explore and analyze geometric shapes, structures, and their properties (11.1)

Demonstrate an understanding of concepts and practices related to data analysis, statistics, and probability (12.2-12.3)

Apply and use measurement concepts and tools (13.1-13.2)

CCCT

[Connecticut Common Core of Teaching]

DEMONSTRATIONS OF KNOWLEDGE

- 1.1 understanding of student learning & development
- 1.2 understanding of need for different learning approaches
- 1.3 proficiency in reading, writing and mathematics
- $1.4\ understanding\ of\ central\ concepts\ \&\ skills,$ tools of inquiry and structures of discipline(s)
- 1.5 knowledge of how to design and deliver instruction
- 1.6 recognition of need to vary instructional methods

APPLICATION OF KNOWLEDGE THROUGH

- 2.1 instructional planning based upon knowledge of subject, students, curriculum & community
- 2.2 selection and/or creation of learning tasks that make subject meaningful for students2.3 establishment and maintenance of appropriate behavior standards and creation of positive learning environment
- 2.4 creation of instructional opportunities supporting students' academic, social and personal development
- 2.5 use of verbal, nonverbal and media communication fostering individual and collaborative inquiry
- 2.6 employment of various instructional strategies in support of critical thinking, problem solving and skills demonstration 2.7 use of various assessment techniques to evaluate student learning & modify instruction

DEMONSTRATION OF PROFESSIONAL RESPONSIBILITY THROUGH:

- 3.1 professional conduct in accordance with the Code of Professional Responsibilities for Teachers
- 3.2 shared responsibility for student achievement and well-being
- 3.3 continuous self-evaluation regarding choices & actions on students and school community
- 3.4 commitment to professional growth
- 3.5 leadership in the school community
- 3.6 demonstrations of a commitment to students and a passion for improving the profession

Quantitative Reasoning

(Competency)

Purpose

To enable students to recognize, understand, and use the quantitative elements they may encounter in various aspects of their lives, to foster abstract quantitative thought, to build self-confidence, and to appreciate the beauty and power of quantitative reasoning. Increasingly, success in modern life, academic disciplines, and career paths depends upon quantitative reasoning.

Key Elements

- 1) <u>Quantitative Situations</u> Identifying the essential quantitative elements in both routine and novel situations and understanding the relationships between those quantitative elements, and producing mathematical models appropriate for the intended analysis (e.g., writing equation(s) to represent the situation).
- 2) <u>Quantitative Data</u> Representing quantitative information in both technical and common language by using symbolic, graphical, and tabular formats, and drawing correct inferences from quantitative information through the interpretations of such representations.
- 3) <u>Methods</u> Acquiring the tools and methods necessary to resolve both routine and novel quantitative questions, including a correct sequencing of procedures, and using them appropriately, given the nature and constraints of a situation. In addition to using knowledge previously acquired in intermediate algebra, students will demonstrate proficiency with information presented in numerical or statistical form and mathematical concepts of growth and decay with their applications (e.g., linear, quadratic, exponential, etc.).
- 4) <u>Reliability of Data and Solutions</u> Correctly evaluating the level of accuracy stated or implied for given data, and assessing the correctness and accuracy of an analysis, including the assessment of the method and model used and the reasonableness of the solution.
- 5) <u>Mathematical Process</u> Using discovery (e.g., exploration and pattern-recognition), conjecture, and testing to develop mathematical formulas, theorems, and then giving persuasive mathematical arguments to establish their validity.

XIV. MAT 105 and MAT 106 Bibliography.

Billstein, R., Libeskind, S. & Lott, J. <u>A Problem Solving Approach to Mathematics for Elementary</u>

<u>School Teachers</u>, 10th Ed., Pearson Addison-Wesley, 2010.

Burns, M. (1992). <u>Math and Literature (K-3) Book One</u>. Math Solutions Publications: Sausalito, CA. Bresser, R. (1995). Math and Literature (4-6). Math Solutions Publications: Sausalito, CA.

Cathcart, W., Pothier, Y., Vance, J. & Bezuk, N. (2006). <u>Learning Mathematics in Elementary and Middle Schools: A Learner-Centered Approach</u>, 4th edition (multimedia edition). Pearson Merrill Prentice Hall: Upper Saddle River, NJ.

Dolan, D., Williamson, J., & Muri, M. (2000). <u>Mathematics Activities for Elementary School</u>

<u>Teachers: A Problem-Solving Approach</u>, 2nd Ed. Addison Wesley: Boston.

Krause, M. (2000). Multicultural Mathematics Materials, 2nd Ed. NCTM: Reston, VA.

National Council for Teachers of Mathematics (NCTM). <u>Principles and Standards for School</u>

<u>Mathematics Navigations Series</u>. [Activity books on probability, data analysis, numbers and operations, algebra and geometry and measurement.] Author: Reston, VA

NCTM (2000). Principles and Standards for School Mathematics. Author: Reston, VA.

NCTM (1992). <u>Addenda Series for Grades K-6</u> [several books that include Developing Number Sense, Dealing with Data and Change, Geometry, Patterns and Functions and Rational Numbers and Proportions. Books are available by grade level.] Author: Reston, VA.

NCTM (ongoing). Yearbook series. [includes titles such as: <u>Making Sense of Fractions, Ratios, and Proportions: 2002 Yearbook</u>,

Mathematical Association of America (MAA) (1991). <u>A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics</u>.

Moore, D. & McCabe, G. (2003) *Introduction to the Practice of Statistics*, fourth edition, pp. 617-618 [for Simpson's Paradox]. W.H. Freeman: New York.

Schifter, D., Bastable, V., & Russell, S. (1999). Developing Mathematical Ideas. [This series has several options including <u>Building a System of Tens, part 1</u>, <u>Making Meaning of Operations, part 2</u>.] Dale Seymour publications: Parsippany, NJ.

Sheffield, S. (1995). <u>Math and Literature (K-3) Book Two</u>. Math Solutions Publications: Sausalito, CA.

Stenmark, J., Thompson, V & Cossey, R. Family Math. Lawrence Hall of Science: Berkeley, CA.

Thompson, V & Mayfield-Ingram, K. <u>Family Math – The Middle School Years</u>. Lawrence Hall of Science: Berkeley, CA.

XV. Website References. The easiest way to find these is to Google the name of the site, but the current websites are listed as well.

National Library of Virtual Manipulatives – choose the topic and/or the grade level

http://nlvm.usu.edu/

IMAP videos

http://www.sci.sdsu.edu/CRMSE/IMAP/main.html

Arcytech

http://arcytech.org/java/b10blocks/http://arcytech.org/java/

NCTM Illuminations web sites

http://illuminations.nctm.org/WebResourceList.aspx?Ref=2&Std=0&Grd=0

Base 5 blocks – paper version for students

http://mason.gmu.edu/~mmankus/Handson/base5.htm

XVI. Prepared.

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XVII. Preparer.

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