BÉZIER CURVES: AN INVESTIGATION INTO MATHEMATICAL CURVES AND HANDWRITING

BY

RACHAEL IVISON

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This honors thesis was prepared under the direction of the candidate's thesis advisor, Dr. Therese Bennett, Department of Mathematics and it has been approved by the members of the candidate's thesis committee. It was successfully defended and accepted by the University Honors Thesis Committee.

Dr. Therese Bennett
Thesis Advisor/Department Chairperson

Dr. Len Brin
Department Reader

Dr. John DaPonte
University Reader

Date

ABSTRACT

Author: Rachael Ivison

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The goal of this research is to produce characters for a font from a sample of handwriting. The font is created using simple JAVA programs and Bézier curves. This paper offers an introduction to Bézier curves and their properties, as well as a motivation for this research through history and applications. The de Casteljau algorithm is explained as a means of creating Bézier curves and B-splines are introduced. Using JAVA, a boundary curve is first extracted from a scanned image of a handwritten character, high curvature points are marked as corners, and tangent vectors are used to calculate intermediate control points. MATLAB is then used to implement the de Casteljau algorithm and to create B-splines that generate the characters for the font. Continuing challenges and possibilities for future research are explored.

Contents

1 Introduction and History

There was a time when the best approximation to a smooth curve was drawn by the human hand. Over the centuries, man has found the need to draw curves that fit more exact specifications. With the advent of computers and computerized machinery, it became increasingly important for mechanical drawings and blueprints to be accurately depicted with a very small error tolerance. In particular, when making automotive parts using mechanical stamps, if the curves were not created precisely, the pieces would not fit together properly. In the 1960s and 70s, there were a handful of mathematicians in the automotive industry working on creating a computer algorithm to address this issue.

Pierre Bézier designed a curve that is created through recursive linear interpolation of pairs of points. The final set of points models the curve. At the time of Bézier's discovery, he was working at the French company Rénault. Another mathematician, Paul de Casteljau, worked at another French company Citroën and developed an algorithm similar to Bézier's. Rénault and Citroën were competing companies and therefore did not want their techniques to be readily available to others. Bézier was allowed to publish anyway so he received credit for the first solution [3]. Thus, these curves are named Bézier curves. The mathematical nature of Bézier and de Casteljau's solution allowed computers to be programmed to create the curves needed to fit certain models.

A Bézier curve is a parametrization of a polynomial. This allows only a few points to be given in order for the polynomial to be found. The given points are called *control points* because their location relative to each other, and the order in which they are interpolated, controls the shape of the final Bézier curve. In order to create the curve, points between a pair of control points are interpolated using a parameter in $\mathbb{R}$, usually named t. The shape of a Bézier curve is only affected by the relative locations of the control points and not by their location relative to their coordinate space. This means that Bézier curves can be discussed in coordinate-free space $\mathbb{E}^3$, as they are in the textbook by Farin [3]. His book describes Pierre Bézier's work along with the work of Paul de Casteljau. It is de Casteljau's algorithm which is described and implemented in this paper. Before explaining the mathematics behind these beautiful curves, a motivation for this project is given.