Individual Round — Arithmetic

(1) What is two-thirds times three-quarters ?

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(2) True or False: If the average of twenty exam scores is 75.65% (and no one actually had a score of 75.65%) there must be ten people who scored higher and ten who scored lower.

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(3) At a summer camp, there are 20 four year-olds, 15 six year-olds, 10 eight year-olds and 5 fifteen year-olds. What is the average age of the campers?

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(4) 2011 is written with exactly 3 different digits. How many years from 1900 to 2011 (inclusive) have exactly 3 different digits?

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Individual Round — Algebra

(1) Solve the linear equation $5x + 2 = 17$.

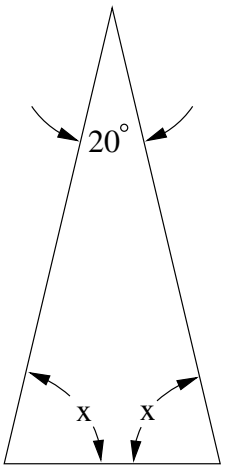
(2) Consecutive perfect squares always differ by an odd number. For example $6^2 - 5^2 = 11$. Which two consecutive squares differ by 2011?

(3) If the graphs of the lines whose equations are $3x - 5y + 2 = 0$ and $2x + Ay - 11 = 0$ are perpendicular, find the value of A.

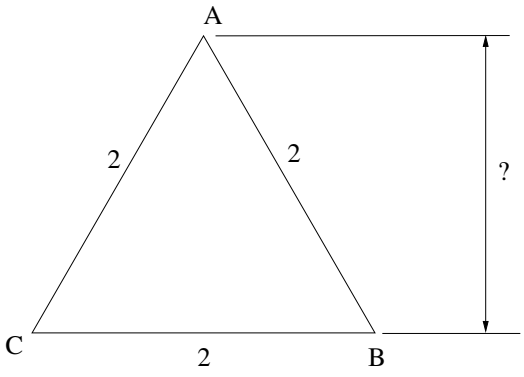
(4) Find all possible solutions of $|x^2 - 12x + 31| = 4$.

Individual Round — Geometry

(1) What is the measure of the angles marked x in the following isosceles triangle?



(2) The triangle $\triangle ABC$ is an equilateral triangle – all of its sides are of length 2. What is its altitude?



(3) Find the value in degrees of the sum of the six angles indicated in the diagram.

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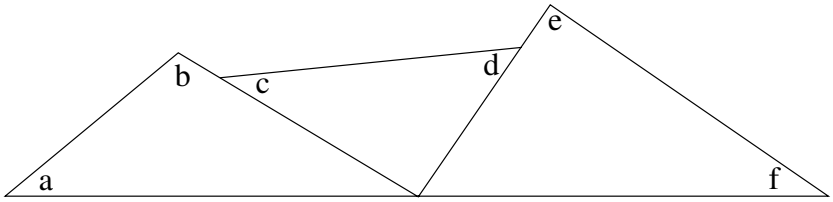
$$a + b + c + d + e + f = \_\_\_\_\_$$



(4) ABCD is a 10cm by 10cm square. Points M and N are placed so that $\overline{BN} = \overline{MD} = 3$. The line through M and N is extended until it intersects the line through A and B. The point of intersection is labeled X. What is the length of segment BX?

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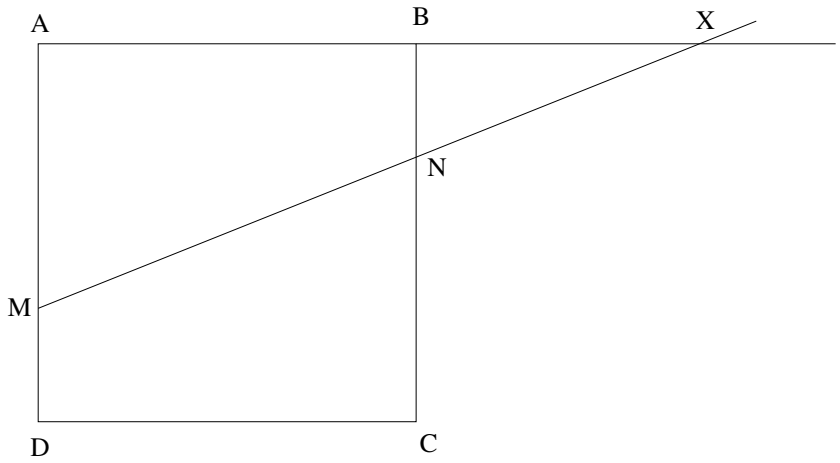

Team Round — CAPT

As part of a promotion, a radio station is giving away 100 smart phones to the first 100 people in line at a concert. To keep it interesting, they decided that 20 of the phones will have a $100 bill slipped in behind the battery, so they will appear to be broken.

(1) Suppose you are the first person selected to receive a phone. What is the probability that you get a "broken" phone?

(2) Suppose you are the 50th person to receive a phone and you know that 8 people who went before you have found $100 bills in their phones. What is the probability now that you get a "broken" phone?

(3) After all the phones have been given out a friend tells you that both she and her cousin got phones. (For this question, assume that you have no further information – you don't know where they were in line, etc.) What is the probability that at least one of them got a phone with a $100 bill?

(4) You went to the concert as part of a friend's birthday party, there was a total of 6 of you, and naturally you were all near the front of the line so all six got phones. What is the probability that *none* of you got a $100 bill?

Team Round — General

(1) Given that $5^n + 5^n + 5^n + 5^n + 5^n = 5^{25}$, find n.

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(2) Two circles are inscribed within a larger circle of diameter 10cm so that they are tangent to one another and to the outer circle, and so that their centers are collinear. What is the sum of the circumferences of the 3 circles?

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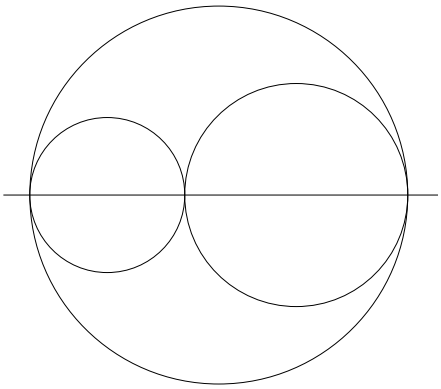

(3) How many zeros are there at the end of $8^{26} \cdot 5^{79}$?

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(4) Triangle $\triangle ABC$ has its vertices at $(0, 6)$, $(8, 0)$ and $(0, 0)$.The points X, Y and Z are the midpoints of segments BC, AC and AB respectively. The line segments AX, BY and CZ are known as the *medians* of the triangle. It is a remarkable fact that the medians all intersect in a single point. What are the coordinates of this point of intersection of the medians (known as the *centroid* of the triangle)?

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