

Individual Round — Arithmetic

- (1) Recall that the *factorial* of a number is denoted using an exclamation point, its value is the product of all the integers from 1 up to that number (for example, $4! = 1 \cdot 2 \cdot 3 \cdot 4$).

Solve for N if

$$6! \cdot 7! = N!.$$

- (2) Find the exact value of $(111, 111, 111)^2$.

- (3) How many positive factors does 900 have?

Individual Round — Algebra

(1) Find all real solutions of $|x - 1| - 4 = 2$.

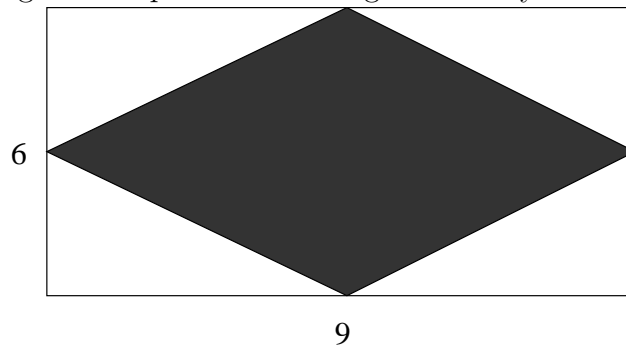
(2) Find all real numbers x , such that

$$2^{x^2-9x-52} = 1.$$

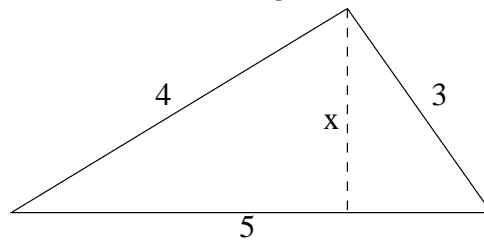
(3) Find a given that 5 is the remainder when $2x^3 + x^2 - ax - 1$ is divided by $x - 2$.

Individual Round — Geometry

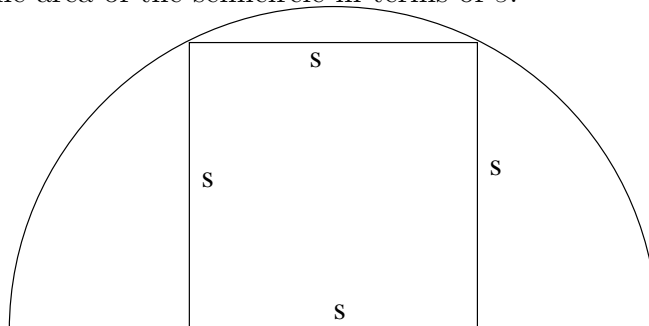
- (1) Find the area of the quadrilateral (shaded in the figure below) formed by joining the midpoints of the edges of a 6 by 9 rectangle.



- (2) Find the altitude, x , of the triangle below.



- (3) A square of side length s is inscribed in a semicircle as shown below. Find the area of the semicircle in terms of s .



Team Round — CAPT

Bill and Melinda got a digital camera as a gift when their baby was born. They know they'll be taking lots of pictures and they'll need to make a lot of copies for family and friends.

Bill finds a service online from Kodak where 4 by 6 inch prints cost 15 cents each and shipping is \$2.00 per order of up to 100.

Melinda thinks they should get their own photo printer. She finds a printer that costs \$199.95 and the best deal on photo paper and an ink cartridge is a kit costing \$36.99 that will yield 160 prints.

- (1) What is the least amount they could spend to get 1000 prints under Bill's plan?

- (2) How much would 1000 prints cost if instead they follow Melinda's plan?

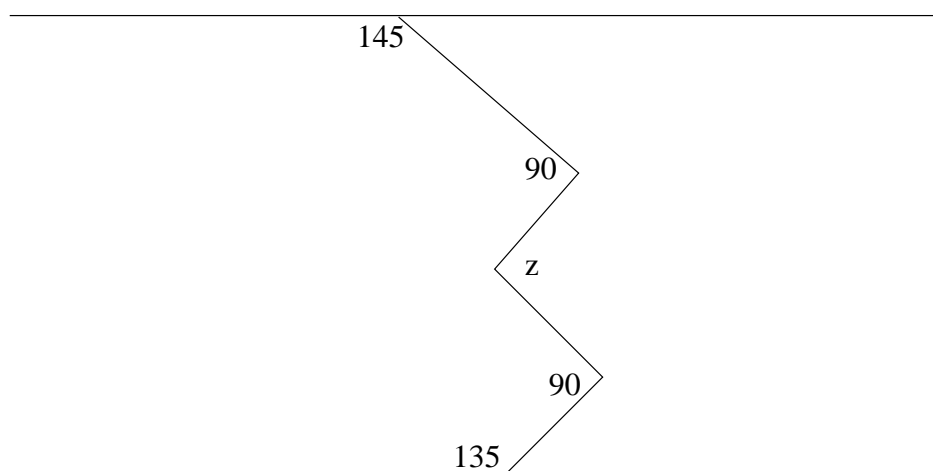
- (3) Note that shipping costs \$.02 per print, and the expense of photo paper and ink averages \$.23 per print. Give formulas for the cost of x prints, calculated on a per print basis, for both schemes.

- (4) After much argument, Bill finally admits that the added convenience of having their own printer is worth something. He tells Melinda that if the overall cost of buying the printer and supplies is no more than double the cost of the online service they can go with her idea. Of course this calculation depends on how many prints they will need. . . What is the least number of paper and ink cartridge kits that they would need to go through in order to justify buying the printer?

Team Round — General

- (1) The number **2006** is a 4 digit number having 2 zeros and two non-zero even digits. How many 4 digit numbers besides 2006 can also be described this way?

- (2) Find the angle z in the diagram below. (The lines at the top and bottom of the diagram are parallel, and the angle measures are in degrees.)



- (3) Find x and y if

$$2^{x+y} = 128 \quad \text{and} \quad 32^{x-y} = 512$$

- (4) How many different triangles can be constructed by choosing three vertices from among the corners of a unit cube?

How many of these triangles have different shapes. That is, how many non-congruent triangles are there?