MAT 375 - Abstract Algebra I

I. Catalog Description

Introduction to the study of algebraic structures. Topics include binary operations, rings, ideals, quotients, homomorphisms, isomorphisms, groups, cyclic groups, the symmetric group, Lagrange's Theorem, normal subgroups and quotient groups.

II. Credit

MAT 375 carries three (3) semester hours of college credit.
MAT 375 is required of majors pursuing a B.S. with 7-12 Certification or a B.A. It is a proof-based elective for the B.S. Applied (two courses among MAT 375, 450 and 480).

III. Prerequisite

The student must have passed MAT 250 and MAT 372 with a C- or better in each.

IV. Format

MAT 375 is primarily a lecture-based course.

V. Outline

- Properties of integers
- Modular arithmetic
- Definition and elementary properties of a ring
- Main examples of rings including number systems (e.g. Gaussian integers), matrices, polynomials, functions, etc
- Various types of rings (including integral domain and field)
- Subring, direct sum of rings, types of elements (e.g. unit, zero-divisor, idempotent, nilpotent)
- Ring isomorphism and homomorphism
- Ideals, PID, quotient rings, and correspondence of ideals and kernels
- Ring Isomorphism Theorems
- Prime and maximal ideals (Optional)
- Definition and elementary properties of a group
- Main examples of groups coming from geometry (e.g. the dihedral groups D_{2n}), number systems (including complex numbers), sets, functions, modular arithmetic, matrices, etc
- Subgroups (including the center, centralizers, conjugate subgroups), subgroup tests, order of elements, direct products
- Cyclic groups and their properties
- Group isomorphism and homomorphism
- The symmetric group S_n and alternating group A_n
- Lagrange's Theorem
- Normal subgroups and quotient groups

VI. Proposed Text

Hungerford, Thomas, Abstract Algebra, An Introduction, Third Edition, Brooks/Cole (Cengage), 2013.

Recommended coverage (excluding appropriate review): Chapters 1, 2, 3, Section 4.1, Chapters 6, 7, Section 9.1, Sections 8.1-8.3.

VII. Other Recommended Texts

Gallian, Joseph, Contemporary Abstract Algebra, Eighth Edition, Brooks/Cole, 2013.

Fraleigh, John, A First Course in Abstract Algebra, Seventh Edition, Addison-Wesley, 2003.

Rotman, Joseph, A First Course in Abstract Algebra with Applications, Eighth Edition, Prentice-Hall, 2006.

VIII. Outcomes

Upon completion of MAT 375, students should be able to demonstrate satisfactory knowledge of the major concepts of abstract algebra, and they should be able to construct symbolically accurate and mathematically correct proofs of basic facts in elementary group, ring and field theory. In particular, students should:

1. Possess a basic but solid knowledge of mathematical objects encountered during their undergraduate math career thus far (such as numbers, matrices, polynomials, functions, sets) and recognize the algebraic similarities they share.

2. Understand the concept of binary operation on a set and the idea of an algebraic structure (a set endowed with operations that satisfy certain axioms).

3. Be able to prove elementary facts about rings and groups by logically combining definitions and theorems.

4. Make connections between groups and familiar geometric objects such as a regular n-gon, tetrahedron, cube and so on (via the dihedral and symmetric groups).

5. Acquire knowledge about the ways to obtain new algebraic structures out of old ones (e.g. sub-structures, quotient structures, direct products).

6. Learn new ways of denoting familiar objects (e.g. cyclic notation for permutations).

7. Recognize when two algebraic structures are abstractly "the same" (in other words, internalize the concept of isomorphism).

IX. Waiver Policy

Course credit or waiver by examination are available for this course.

X. Preparers

Alain D'Amour & Jooyoun Hong

XI. Prepared and Approved

Prepared on April 5, 2016. Approved by the MDCC/Math Dept on