GEOMETRY AND THE ARTS
TIER 2 – CREATIVE DRIVE

I. Course Information

- Department: Mathematics
- Course Code and Number: MAT 260
- Number of Credits: 3 credits
- Course Title: Geometry and the Arts
- Course Description (maximum of 100 words): The purpose of the course is to provide students with an adequate geometric background allowing them to understand basic Euclidean geometry. The course will enable students with basic tools for geometric reasoning and proof and will promote their geometric thinking skills.

  Throughout the course, students will explore and appreciate the use of geometry in the arts. They will identify, analyze, replicate and produce art work using geometric figures and transformations.
- Prerequisite:
  - For students in LEP: Tier 1 - Critical Thinking AND MAT 103 or higher.
  - For students who are not in LEP: MAT 103 or higher.
- Mode of instruction: The course will provide students with hands-on/minds-on experience using manipulatives, calculator applets and dynamic geometry software. Students are expected to participate in class activities and discussions, to complete homework assignments and projects, and to present them in class.
- Technology: A graphic calculator (TI-83 plus or TI-84 plus) is required
- Date course to be first offered: Fall 2012
- Required Textbook:
  - Musser, Trimpe & Maurer (2008). *College Geometry: A Problem Solving Approach with Applications – 2nd edition.* Prentice Hall
- Recommended Textbook:
  - Marc Frantz & Annalisa Crannell (2011). Viewpoints: Mathematical perspective and fractal geometry in Art. Princeton Press.

- Reference Textbooks:
  - Thomas L. Heath (1956). *The **Thirteen Books of Euclid's** Elements.* Translation. Dover Publications
  - David Hilbert (1971). Foundations of Geometry. Translated by Leo Unger from the German ed. (10th ed) Open Court
  - George David Birkhoff & Ralph Beatley (1959). Basic Geometry. Chelsea
  - Catherine Gorini (2000). *Geometry at work*. Cambridge University Press
  - Claude Bruter (2002). *Mathematics and Arts*. Springer
  - Robert Field (1997). *Geometric Patterns from Churches and Cathedrals.* Tarquin
  - Paulus Gerdes (1999). *Geometry from Africa– Classroom Resource Materials.* The Mathematical Association of America Publishers
  - Andrew H Whiteford (2001): *North American Indian Arts*. Golden Guides
  - Paulus Gerdes (1998). *Women, Art and Geometry in South Africa*. Africa World Press
  - Daud Sutton (2007). *Islamic Design: A genius for Geometry*. Walker & Company
  - Moriteus Cornellis Escher (1983): *29 Master Prints.* Harry Abrams.
  - Hajime Ouchi (1977): *Japanese Optical and Geometrical Art.* Dover Publications

II. Rationale for Course Proposal

This course is proposed to satisfy the Creative Drive component in tier 2 of the Liberal Education Program. It will explore the creative drive through the production and presentation of an original work based on geometry. Many sculptures, architectural work, paintings and drawings are based on geometric shapes and geometric transformations. Thus, these artistic pieces provide an external motivation for the study of geometry and conversely, geometry deepens the understanding of these works of art.

In addition to the creative process, geometric thinking and reasoning foster several thinking and reasoning skills that students use in areas other than geometry. However, many students did not have enough experience with high school geometry and have difficulty in this content area. The

need for a new geometry course that deepens basic geometry content and prepares students to construct implicit geometric proofs, as distinct from explicit proofs, was identified in the Math department. Thus, in addition to addressing Creative Drive, this course will help to prepare elementary and middle school teachers for teaching geometry and will ensure a proper foundation for students who plan to take Euclidean and Non-Euclidean Geometry, MAT 360. This course will be proposed to all students who need to strengthen their basic geometry knowledge, and will be recommended to Elementary Education and to Math Education Majors. The course is expected to run once a year, at least for one section of 20 students.

III. Course overview

The purpose of this course is to connect geometry with the arts by teaching the basics of geometry while revealing the mathematical knowledge that is embedded in art works.

Using a problem-based approach, and hands-on/minds-on activities, students will acquire essential geometric knowledge and learn how to use it to construct, justify and validate geometric constructions.

Students will be exposed to several artistic productions where geometric shapes and transformations are embedded in the work (for example: work of M.C. Escher, Roses in Medieval Churches, Mosaics and Islamic art, Patterns in quilts and texture (African, Indian and Mayan arts, North American Indian Arts), Japanese arts, architecture (Roman Coliseum, Pyramids)) (*meeting Key elements 1 and 4 in Creative Drive Course*).

At the beginning of the course, students will imitate and replicate given figures (simple to complex), and at the end they will be producing a new production. In order to do this, they will learn how to manipulate geometric tools (compass, ruler, and protractor) and dynamic software to create drawings and models (*meeting Key element 2 in Creative Drive course*). In order to support their claim to have met the objectives of each project, they will learn how to use geometric knowledge to explain, justify and prove given facts and dynamic geometric software to validate them. At the end of the course, students will be expected to be able to write a two column geometry proof (*Critical thinking, Creative thinking and written communication competencies*)

Students will present their projects in a poster session in class and then in the Math Department (*meeting Key Element 3 in a Creative Drive Course*). They will have to explain the steps they have used, the calculations they have done, the geometric knowledge they have used and the justification of their work.

IV. Learning Objectives *(Key elements of Creative Drive are indicated by CD)*

Upon completion of this course, students will be able to:

1. Use appropriate vocabulary and terminology to describe geometric figures and solids.
2. Build and manipulate representations of two- and three- dimensional objects and visualize them from different perspectives (NCATE 11.4 , CD 1, 2, 4)
3. Use concrete models, drawings, and dynamic geometric software to explore geometric ideas and their applications in real-world contexts (NCATE 11.7, CD 2)
4. Analyze art work (3D model or 2D drawing) from a geometrical point of view by identifying all mathematical objects, shapes and transformations being used and writing the corresponding program of construction (NCATE 11.3 , CD 4)
5. Replicate, enlarge and create drawings and models based on geometric shapes and transformations, using concrete tools and dynamic geometry software (NCATE 11.6 , CD2,4)
6. Use dynamic software and/or calculator application to make and validate geometric conjectures.
7. Use geometric properties and theorems to infer segment or angle measures (polygons and circles) to justify or validate constructions and to solve problems (CD 1, 2, 4)
8. Produce and communicate written deductive proofs to explain and validate a construction or a result using theorems and properties (CD 3)
9. Produce and present an original geometry and art inspired project with written explanation (CD 3)

V. Meeting LEP Tier 2 Creative Drive Requirements

Multiple instructors may be involved in the instruction of this course, it is expected that the art work exposed in this course will vary from one instructor to another. However, all sections must, at a minimum, meet the requirements of creative thinking as an embedded competency in the course and the aesthetic sensitively value.

1. Area of knowledge: "Creative Drive"
   Through study of geometry and art, students will acquire a set of tools to analyze and execute works of geometric art. The course will culminate in presentation of the student's final art project.
2. Reinforced competency (additional):
   "Creative thinking" is an essential part of the creative process. Throughout the course, students will be asked to design and execute art work, to make and test conjectures and to prove or negate statements. Students will write coherent justifications to accompany their work and to solve problems. They will be asked to write a complete structured program allowing them to construct complex geometric figures.

   In addition, technology is used in significant and extensive ways in this course as a means through which students will make and verify conjectures and produce drawings.
3. Discussion of values: Aesthetic sensitivity
   Examples from arts and architecture from various cultures will be used as examples of using geometry. This will enable students to appreciate the importance of Geometry in Art and they will be exposed to different cultures through architectural work and ornamentation. They will discuss the development of criteria as they assess the work of other class members.
4. Intellectual Foundations Component: Students will compare and contrast the treatment of various components of geometric theory from seminal works of Euclid, Birkhoff and Hilbert.

- Thomas Heath (1956). **The Thirteen Books of Euclid's Elements**, Dover
- George David Birkhoff & Ralph Beatley (1959). Basic Geometry, Chelsea
- David Hilbert (1902), The Foundations of Geometry, Open Court.

VI. Assessment

Individual instructors may vary assessment modes, but typically grades will be based on a combination of class activities, homework assignments (papers and projects), quizzes, a final project with poster presentation, peer assessment and a final written exam, distributed as follow:

- Class activities: (20%)
    - Students are expected to submit a worksheet and a product at the end of their work.
        - Technology based activities (software or calculator application),
        - Hands-on / Minds-on activities
        - Group activities
- Homework assignments: (20%)
    - Written assignments (mini papers, problem solving, construction or drawings accompanied with explanations)
- Quizzes: (20%)
    - Written, announced quizzes
- Final exam: (20%)
    - Geometry content covered in the course
- Final project and presentation (20%)
    - Students are expected to create an original art project, to present its construction in a poster session and to assess fellow students' work.

VII. Course outline

The table on pages 7-10 represents a sample of teaching and assessment activities for each of the course objective.

The table on pages 11-16 represents a tentative course outline. Column 1 shows the week number in the semester. Column 2 shows the topics to be covered during that week. Column 3 shows the Creative Drive Key elements met through the learning of that topic and the LEP embedded competencies that students will reinforce. Column 4 shows the Learning activities and Column 5 shows the assessment activities associated with the topic of the corresponding week.

TEACHING AND ASSESSMENT ACTIVITIES

Learning Goals	Teaching / Learning Activities	Assessment activities
1. Use appropriate vocabulary and terminology to describe a geometric figure or solid.	Group activity / interactive lecture • Group activity: Students work in groups to identify the geometric elements and transformations embedded in a given art work. Teacher follows up on the activity with an interactive presentation about the proper terminology for basic elements in geometry.	Mini Papers , Assignments and quizzes • Write one-minute paper(s) using geometric vocabulary and terminology to describe an object /art work • Throughout the assignments of this course, students are required to use correct and appropriate terminology to describe geometric shapes and transformations
2. Build and manipulate representations of two- and three- dimensional objects and visualize them from different perspectives.	Hands-on / minds-on activities followed by interactive lecture: • Group activity: Students are asked to create replicas of a given figure using different procedures and different geometry tools (compass, ruler and protractor) • Students explore 3D models, build their nets, identify their elements and compare them to their 2D drawings. Technology based activities: • Students explore 3D geometric objects from different perspectives, using dynamic computer Software (Geospace, Cabri 3D) and construct 2D drawings of these objects.	Assignments and quizzes: • Create figures (replicas and new) of given 2D figures using concrete material and using dynamic software • Create models (replica and new ones) of 3D models using concrete material and using dynamic software • Construct 2D drawing for solids using paper and pencil

3. Use concrete models, drawings, and dynamic geometric software to explore geometric ideas and their applications in real-world context	Hands-on / minds-on activities followed by interactive lecture: • Students create replica of architectural models or art models using concrete material. • Group activity: Given a picture of a real-world model, students describe all geometric elements embedded in the picture and identify all the elements that differ in the drawing from the reality of the object. • Students construct a 2D drawing of a real world (3D) object (pictures of architectural work, pyramids, Greek columns, etc…) Technology based activity: • Students build geometric models using dynamic software (Sketchpad, Geospace or Cabri 3D)	Assignments: • For an architectural model of their choice, students create a concrete replica of the model and a drawing using dynamic software. • For a given art work, students create replicas using concrete material and/or using dynamic software
4. Analyze an art work (3D model or 2D drawing) from a geometrical point of view by identifying all mathematical objects, shapes and transformations being used and writing the corresponding program of construction.	Problem-based activities / Group activities: • Group activity: Students are asked to outline the geometric shapes and transformations that are embedded in a given art work (work of Escher, Rosettes, Mosaics, etc...). Then, they use these steps to validate their answers. • Students are asked to write the chronological steps needed to construct a given model or drawing. • Activities followed by class discussion and teacher intervention.	Assignments, class activities and quizzes • For a drawing or model of their choice, students explain the cultural aspects of the work of art and then outline all geometric objects and transformations used in the work, using proper terminology. • Given a drawing or model, students are asked to write the appropriate program of construction leading to the given figure • Quiz and exams

5. Replicate, enlarge and create drawings and models based on geometric shapes and transformations, using concrete tools and dynamic geometry software.	Problem-based activities / Group activities / Interactive lecture • Group activity and presentation: Given a complex geometric figure, students are asked to each enlarge a piece of it. If the group use an appropriate strategy than the final product is a correct enlargement of the figure. Students present their work and strategy to class. • Through interactive lecture, students are asked to (then taught how to) replicate and create figures based on transformations (Fractals, tessellations, ,,,) • Students work in pairs to explore (and later create new) plane tessellations. Technology based activities: • Students create plane tessellations using dynamic software • Students explore geometric transformations using dynamic software or Calculator applications.	Assignments, quizzes and final project • Group work: students create replica and enlargement of a given figure and explain the work • Group work*:* students create replica and enlargement of a given figure using dynamic software and explain the work. • Students choose an art work that uses transformations or tessellations, explain its cultural context and present a replica of the work. • Students are asked to create a new "form" that tessellate the plane, explain the validity of the work while presenting their art work • Quiz and exams
6. Use dynamic software and/or calculator application to make and validate geometric conjectures.	Technology based activities (lab or class activities) • Using dynamic software or calculator application, students make conjectures to answer a given question. They use technology to test their conjecture and modify it (or to validate it).	Lab activities • Students fill out report worksheet of their lab activities

7. Use geometric properties and theorems to infer segment or angle measures (polygons and circles) to justify or validate constructions and to solve problems.	Hands-on/minds-on activities: • Using geometry tools, students construct basic geometric figures (special lines in a triangle, quadrilaterals, polygons, circles, …) • Group activity: Using geometry tools and geometric properties, students are asked to find different valid ways to construct polygons • Pair work: Each student writes the steps needed to construct a given figure; their partner follows their instruction to construct the figure. Activity followed by class discussion. • Group work: After being presented with a drawing, students are asked to analyze it in order to find the constitutive elements and their measures then write a program of construction. Problem-based activities followed by interactive lecture: • Group activity: Students are asked to use geometric properties to classify quadrilaterals, angles, triangles, polyhedrons, etc … • Problem solving: Using theorems, students are asked to calculating measures (sides, angles, central angles of a circle …) and to justify their work using theorems and properties. Activities followed by interactive discussion. • Solve problems involving properties learned and write simple proofs.	Assignments, quizzes and final project • Solving problems involving properties of triangles to calculate lengths of sides or angles measures. • Solving problems involving properties of quadrilaterals and writing simple proofs. • Solving problems involving measure of central angles or interior angles of a circle. • Construct replicas of simple figures involving quadrilaterals, triangles, circles and write programs of construction • Given a figure and its replica or enlargement, students need to validate the given dimensions using their knowledge of properties and theorems • Quiz and exams
8. Produce and communicate written deductive proofs to explain and validate a construction or a result using theorems and properties.	Problem-based activities and Interactive lecture: • Use geometric theorems and properties to justify and prove given statements. • Solve geometry problems involving writing justifications.	Homework, Quiz, final exam • Solving problems involving congruence of triangles, theorems, properties and writing simple proofs.
9. Produce and present an original geometry and art inspired project with written explanation	Problem-based activities • Throughout the course, students will learn how to use geometric tools and technology along with geometric knowledge in order to create an original art project.	Final project • Throughout the course, students will develop their final original art project and they will present it in a peer reviewed poster session.

COURSE OUTLINE AND ASSIGNMENT SCHEDULE

Week	Topic	LEP requirements addressed	Learning Activities	Assessment Activities
Week 1	Introduction, Syllabus Basic Tools for geometry (point, line, plane, shapes)	Values Discussion Aesthetic Sensitivity Reinforced competency: Written Communication	• Group activity: Students work in groups to identify the geometric elements and transformations embedded in a given art work. Teacher follows up on the activity with an interactive presentation about the proper terminology for basic elements in geometry	• Write one-minute paper(s) using geometric vocabulary and terminology to describe an object /art work
Weeks 2, 3	Manipulation of compass, protractor and rulers to replicate and construct basic simple figures. Circle, Midpoint of a segment, perpendicular bisector Linear Measurement, angle measurement	Reinforced competency: Creative thinking	Hands-on/ Minds-on activities / Interactive discussion • Students are asked to replicate a given figure or model using geometry tools (compass, ruler, and protractor). • Group activity: Students are asked to create replicas of a given figure using different procedures and different geometry tools (compass, ruler and protractor) Problem-based learning • Solving problems involving converting linear measures or calculating angle measures or length using geometric properties. Technology based activities: • Students construct geometric figures and models using dynamic computer Software (The Geometer Sketchpad) or Calculator App (Cabri Jr)	• Create figures (replicas and new) of given 2D figures models using concrete material and using dynamic software • Class activities, problem solving • Quiz

Week 4	Triangles: classification of triangles, special lines in a triangle (perpendicular bisector, angle bisector…) and their properties.	Reinforced competencies: Critical thinking Creative thinking	Problem-based activities: • Solving problems involving properties of triangles to calculate side measures or angle measures. Hands-on/minds-on activities: • Using geometric tools, students learn how to construct special lines in a triangle • Group activity: Using geometric tools and properties, students are asked to find different ways to construct a given figure. Technology based activities: • Solve problems involving properties of special lines in a triangle and writing simple proofs	• Construct replicas of simple figures involving triangles and write a program of construction • Solve problems involving properties of triangles, lengths of sides and measures of angles. • Class activities, problem solving • Quiz
Weeks 5,6	Quadrilaterals: Classification of quadrilaterals and properties	Reinforced competencies: Critical thinking Creative thinking	Hands-on/minds-on activities: • Group activity: Using geometric tools and properties, students are asked to find different ways to construct a given quadrilateral. • Pair work: Each student writes the steps needed to construct a given figure; their partner follows their direction to construct the figure. Activity is followed by class discussion. • Group work: After being presented with a drawing, students are asked to analyze it in order to find the constitutive elements and their measures and then write a program of construction. Technology based activities: make and verify conjectures Problem-based activities: • Solve problems involving properties of quadrilaterals and writing simple proofs	• Construct replicas of simple figures involving quadrilaterals and write a program of construction • Solving problems involving properties of quadrilaterals and writing simple proofs • Class activities, problem solving • Quiz

Week 7	Solids: Classification, drawings	Reinforced competencies: Critical thinking Creative thinking	Hands-on / minds-on activities followed by interactive lecture: • Students explore 3D models, build their nets, identify their elements and compare them to their 2D drawings. • Group activity: Given a picture of a real-world model, students describe all geometric elements embedded in the picture and identify all the elements that differ in the drawing from the reality of the object. • Students construct a 2D drawing of a real world (3D) object (pictures of architectural work, pyramids, Greek columns, etc…) Technology based activity: • Students explore 3D geometric objects from different perspectives, using dynamic computer Software (Geospace, Cabri 3D) and construct 2D drawings of these objects.	• For an architectural model of their choice, students create a concrete replica of the model and a drawing using dynamic software.

Week 8	Analyzing a complex figure and writing a sequential program for its construction (Properties of polygons and their elements) Create a replica of a complex figure or its enlargement	Key Elements of C.D.: 1) Articulation of Project 2) Development of project Reinforced competencies: Critical thinking Creative thinking	Problem-based activities: • Group activity and presentation: Given a complex geometric figure, students are asked to each enlarge a piece of it. If the group use an appropriate strategy than the final product is a correct enlargement of the figure. Students present their work and strategy to class. • Students are asked to write the chronological steps needed to construct a given model or drawing. • Group work: Given a complex figure, students are asked to analyze it in order to find the needed measures and to write a sequential program of construction.	• Students are asked to choose an art work that uses geometric shapes, explain its cultural context and present a replica of the work • Given a drawing or model, students are asked to write the appropriate program of construction leading to the given figure. • Group work: students create replica and enlargement of a given figure and explain the work • Class activities, Problem solving • Quiz

Weeks 9, 10	Regular and irregular tessellations Transformations (translation, reflection and rotation)	Key Elements of C.D.: 1. Articulation of Project 2. Development of project Reinforced competencies: Critical thinking Creative thinking Written Communication	Problem-based activities / Interactive lecture: • Through interactive lecture, students are asked to (then taught how to) replicate and create figures based on transformations (Fractals, tessellations...) • Group activity: Students are asked to outline the geometric shapes and transformations that are embedded in a given art work (work of Escher, Rosettes, Mosaics, etc...). Then, they use these steps to validate their answers. • Students work in pairs to explore (and later create new) plane tessellations. Technology based activities: • Students create plane tessellations using dynamic software • Students explore geometric transformations using dynamic software or Calculator applications.	• Students are asked to create a new "form" that tessellate the plane, explain the validity of the work while presenting their art work • For a drawing or model of their choice, students explain the cultural aspects of the work of art and then outline all geometric objects and transformations used in the work, using proper terminology.
Weeks 11, 12	Circles, angles in a circle, chords (Examples: Rosette in Medieval Churches or Mosaic in Islamic Art)	Key Element of C.D.: 4. Creative Process and Exemplars Reinforced competencies: Critical thinking Creative thinking Written Communication	Hands-on/Minds-on activity: • Given a figure involving circles and angles, students need to write a program enabling its construction and justifying the steps. Problem-based activities followed by interactive lecturing: • Using theorems and geometric properties in circles, students are asked to calculate measures of angles.	• Class activities and quizzes: Construct replicas of figures involving circles and write program of construction. • Students choose an art work involving circles and arcs, explain its cultural context and present a replica of the work

Week 13	Congruence of triangles, similarity of triangles	Reinforced competencies: Critical thinking Creative thinking	Problem-based activities: • Given a figure and its replica or enlargement, students will validate the given dimensions using their knowledge of properties and theorems • Solving problems based on congruence of triangles and writing a proof	• Class activities and quizzes: Solve problems involving congruence of triangles and writing simple proofs
Week 14	Writing proofs	Reinforced competencies: Critical thinking Creative thinking Written Communication	Problem solving activities: • Use geometric theorems and properties to justify and prove given statements. • Solve geometry problems involving writing justifications Technology based activities: • Using dynamic software or calculator applications, students make conjectures to answer a given question. They use technology to test their conjecture and modify it (or validate it).	• Class activities and quizzes: solving problems involving writing proofs
Week 15	Project poster presentations, Peer- assessment of projects	Key Elements of C.D.: 3. Presentation of project 4. Examining Process and Exemplars Values discussed: Aesthetic sensitivity	• Using geometric tools and technology along with geometric knowledge, students create and present an original art project.	• Poster presentation and peer assessment of the students' original art projects