SOUTHERN CONNECTICUT STATE UNIVERSITY

Mathematics 250
Foundations of Mathematics: An Introduction

I. *Description*

(a) Catalog description: A bridge between calculus and upper level mathematics courses. Logic, sets, relations, functions, methods of proof.

(b) Extended description: The course uses examples from calculus, elementary number theory, geometry, discrete mathematics, basic abstract algebra and linear algebra. Emphasis is on concepts that will be encountered in later undergraduate courses. Essential topics are:

i. Logic (7%)
ii. Methods of Proof (35%)
iii. Set Theory (7%)
iv. Relations (15%)
v. Functions (15%)
vi. Cardinality (7%)

This is a one semester course meeting three contact-hours per week during a regular semester. It is intended for Mathematics majors. Note that the above schedule allows 14% for examinations and review sessions.

II. *Credit*

A. MAT 250 carries three semester-hours of university credit.
B. MAT 250 is required of all Mathematics majors.
C. A student cannot get credit for both MAT 178, and MAT 250.

III. *Prerequisites*

C- or better in MAT 151.

IV. *Purpose*

Mathematics 250 provides an introduction to certain fundamental concepts of mathematics. The concepts are ones that every mathematics major is expected to know and are used in nearly all upper-level undergraduate and graduate mathematics courses. Mathematics 250 is an intermediate-level course lying between the practically-oriented calculus sequence and the theoretically-oriented upper-level courses.

V. *Format*

Lecture.

VI. *Outline*

(a) Logic
   i. Propositions and Connectives
   ii. Conditionals and Biconditionals
   iii. Quantifiers

(b) Methods of Proof
   i. Proving universal statements
      A. Trivial Proofs
      B. Vacuous Proofs
      C. Direct Proofs
      D. Proofs by Contraposition
      E. Proofs by Contradiction
      F. Proofs by Cases
   ii. Disproving universal statements
   iii. Proving existential statements
      A. Constructive Existential Proofs
      B. Non-constructive Existential Proofs
   iv. Disproving existential statements
   v. Proving and disproving statements with both quantifiers
   vi. Proofs using the principle of mathematical induction

(c) Set Theory
   i. Basic notations and operations
   ii. Set identities
   iii. Indexed families of sets
   iv. Power sets

(d) Relations
   i. Properties of relations (symmetry, anti-symmetry, transitivity, reflexivity, etc.)
   ii. Equivalence relations
   iii. Ordering relations

(e) Functions
   i. Terminology (domain, codomain, range, image, preimage, etc.)
   ii. Restrictions, extensions, projections, compositions
   iii. Injectivity, surjectivity, bijectivity

(f) Cardinality of sets and the continuum hypothesis.

VII. *Recommended text*

Douglas Smith, Maurice Eggen and Richard St. Andre, *A Transition to Advanced Mathematics, 5th edition*. Brooks/Cole, 2001.

VIII *Objectives*

Students in MAT 250 should achieve several departmental and program objectives. The objectives below are cross-referenced with the departmental, National Council of Accreditation of Teacher Education (NCATE), Interstate New Teachers Assessment and Support Consortium (INTASC), and Connecticut Common Core of Teaching (CCT) objectives.

Students will be able to:

(a) State and apply logic and set operations, properties of number systems, and properties of functions and relations.
(DEPT 1; INTASC 1; CCT I 3,4; NCATE, 1.4,1.51)

(b) Explore ideas within mathematical structures presented to form, investigate, and prove conjectures.
(DEPT 9; INTASC 1; CCT I 4; NCATE 1.2)

(c) Demonstrate the ability to write mathematical proofs.
(DEPT 3; INTASC 1,6; CCT I 3,4; NCATE 1.2,1.3,1.5.1,1.5.9)

(d) Communicate effectively and explain mathematics both verbally and in writing.
(DEPT 6; INTASC 1; CCT I 3,4, II 5; NCATE 1.3)

(e) Use acquired mathematical skills to undertake independent learning and to be a contributing member of a problem-solving team.
(DEPT 2; INTASC 1; CCT I 3,4; NCATE 1.1,1.2,1.3)

(f) Appreciate the beauty, joy and challenge in mathematics and experience mathematics as an engaging field with contemporary open questions.
(DEPT 8; INTASC 1; CCT I 4; NCATE 1.4)

IX. *Date*

Revised by Alain D'Amour, Joseph Fields, and Val Pinciu, April 2002.

X. *References*

[1] Robert Ash. *A Primer of Abstract Mathematics.* MAA, (Classroom Resource Materials series), 1998. ISBN: 0-88385-709-X.

[2] William Barnier and Norman Feldman. *Introduction to Advanced Mathematics.* Prentice Hall, Englewood Cliffs, NJ, 1990. ISBN: 0134770846.

[3] Donald C. Benson. *The Moment Of Proof: Mathematical Epiphanies.* Oxford University Press, 1999. ISBN: 0-19-511721-2.

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[9] Daniel Fendel and Diane Resek. *Foundations of Higher Mathematics: Exploration and Proof*. Addison Wesley, Boston, MA, 1990. ISBN: 0-201-12587-0.

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[12] Douglas Hofstadter. *Godel, Escher, Bach: an eternal golden braid*. Basic Books, New York, NY, 1979. ISBN: 0465026567.

[13] Imre Lakatos, E. Lakatos, and J. Worrall (Editor). *Proofs and Refutations : The Logic of Mathematical Discovery*. Cambridge University Press, 1977. ISBN: 0521290384.

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[15] John F. Lucas. *Introduction to Abstract Mathematics, 2/e*. Ardsley House, New York, NY, 1990. ISBN: 091267573X.

[16] Joseph J. Rotman. *Journey into Mathematics, A: An Introduction to Proofs*. Prentice Hall, 1997. ISBN: 0138423601.

[17] Carol Schumacher. *Chapter Zero: Fundamental Notions of Abstract Mathematics, 2/e*. Addison Wesley, Boston, MA, 2001. ISBN: 0-201-43724-4.

[18] Douglas Smith, Maurice Eggen, and Richard St. Andre. *A Transition to Advanced Mathematics, 5/e*. Brooks/Cole, Pacific Grove, CA, 2001. ISBN: 0-534-38214-2.

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