1st Annual GeoGebra Conference
CAS Presentation

5 November 2013

CAS as Calculator

Press ≈ button before starting

1. `32+56`
2. `32/56+12/28-5/7`
3. `sin(45°)`
4. `e^3`
5. `π*6^2`

Fire up the virtual keyboard

6. $\sqrt{448}$
7. `80+85+32+75+83`
   (a) `$/5`

Calculator improved—Go back and change one of the numbers in 7.

8. `3^90`
9. `100!`

Go back and recalculate all exactly

CAS as Graphing Calculator

1. `f(x):=-x^2-2x+4`

Color f purple

2. `g(x):=f(x-3)`
3. `h(x):=f(x+1)`
4. `p(x):=f(x-2)`

Color g, h, and p different colors. Then delete f(x)—all will disappear

5. `f(x):=2x^3-5x-8`

Use scroll to zoom out to a nice view and use move tool to center it
Put a point on $f(x)$ and drag it around
Create tangent line through new point
Hmmm...let's plot the derivative function

6. `g(x):=Derivative[f(x)]`
7. `B:Point[{x[A],g(x[A])}]`

Use move tool to scale x-axis only—make a nicer view and drag point around

CAS as Statistical Calculator

Press ≈ button before starting
Select data and create list using button

1. `Min[list1]`
2. `Q1[list1]`
3. `Median[list1]`
4. `xbar:Mean[list1]`
5. `s:SampleSD[list1]`

Change a value or two in the spreadsheet

6. `n:Length[list1]`
7. `tscore:InverseTDistribution[n-1,.975]`
8. `m:tscore*s/sqrt(n)`
9. `CI:(xbar-m,xbar+m)`

Show One Variable Analysis button

CAS Solving Algebra Problems

1. Factor 60. `Factor[60]`
2. Expand $(x+8)(2x-3)$. `(x+8)(2x-3)`
3. Solve $3x - 5 = 12 - 2x$. `Solve[3x-5=12-2x]`
4. Solve $a^2+7^2 = 12^2$. `Solve[a^2+7^2=12^2]`
5. Factor $4x^2-12x+9$. `Factor[4x^2-12x+9]`
6. Factor x^2-3x-2. `Factor[x^2-3x-2]`
7. Complete the square: $x^2 - 3x - 2$. `CompleteSquare[#]`
8. Long divide $4x^3-3x^2+12$ by x^2+2x-1. `Division[4x^3-3x^2+12,x^2+2x-1]`
9. Solve the nonlinear system
$$\begin{cases} x = y^2 - 2y - 1 \\ y = 2 + x \end{cases}$$
`Solve[{x=y^2-2y-1,y=2+x},{x,y}]`

CAS on a Calculus Exploration

1. `f(x):=x³ + RandomBetween[-11,11] x² + RandomBetween[-11,11] x + 1`
2. `g(x):=Derivative[f(x)]`

Press ≈ button before continuing

3. `ComplexRoot[f(x)]`
4. `Mean[$4]`
5. `ComplexRoot[g(x)]`
6. `Mean[$6]`

Rerun line 5 multiple times to see the pattern.

Lemma: Any polynomial $p(x)$ of degree $n \geq 1$ can be written in the form $p(x) = a(x^n - n\overline{r}x^{n-1} +$ lower order terms) for some constant a and $\overline{r}$ equal to the average of its roots.

Proof: Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ be a polynomial of degree $n \geq 1$. Rewrite the polynomial in the form $a_n(x-r_1)(x-r_2)\cdots(x-r_n)$ where $r_1, r_2, \ldots, r_n$ are the n (possibly complex) roots of $p(x)$ and multiply to find that

$$\begin{aligned} p(x) &= a_n(x^n - (r_1 + r_2 + \cdots + r_n)x^{n-1} + \text{lower order terms}) \\ &= a_n(x^n - n \cdot \frac{r_1 + r_2 + \cdots + r_n}{n} x^{n-1} + \text{lower order terms}) \\ &= a_n(x^n - n\overline{r}x^{n-1} + \text{lower order terms}). \end{aligned}$$

Proposition: The average of the the roots of any polynomial of degree 2 or greater equals the average of the roots of its derivative.

Proof: Given a polynomial $p(x)$ of degree $n \geq 2$, note that $p'(x)$ has degree at least 1. By the lemma, we may write $p(x)$ in the form $a(x^n - n\overline{r}x^{n-1} + \text{lower order terms})$ and $p'(x)$ in the form $b(x^{n-1} - (n-1)\overline{s}x^{n-2} + \text{lower order terms})$ where $\overline{r}$ is the average of the roots of p and $\overline{s}$ is the average of the roots of p'. Calculating $p'(x)$ from $p(x)$, we find

$$\begin{aligned} p'(x) &= a(nx^{n-1} - n(n-1)\overline{r}x^{n-2} + \text{lower order terms}) \\ &= an(x^{n-1} - (n-1)\overline{r}x^{n-2} + \text{lower order terms}). \end{aligned}$$

Equating the two expressions for $p'(x)$, we find that $b = an$ so that

$$-(n-1)\overline{s} = -(n-1)\overline{r}$$

from which it follows that $\overline{s} = \overline{r}$.